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The Relationship between Teacher Pedagogical Content Knowledge and Student Understanding of Integer Operations

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The Relationship between Teacher Pedagogical Content Knowledge and Student Understanding of Integer Operations

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The Relationship between Teacher Pedagogical Content Knowledge and Student Understanding of Integer Operations

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The purpose of this study was to determine whether a professional development (PD) for teachers focused on improving teacher pedagogical content knowledge (PCK) related to operations with integers would improve teacher PCK and if there was a relationship between their level of PCK and the change in the understanding of their students as measured by pre- and posttest of teacher and student knowledge. The study was conducted summer 2010 in a large urban school district on two campuses providing a district funded annual summer intervention, called *Jumpstart*. This program was for grade 8 students who did not pass the state assessment (Texas Assessment of Knowledge and Skills), but would be promoted to high school in the Fall 2010 due to a decision made by the Grade Placement Committee. The Jumpstart program involved 22 teachers and 341 students.

For purposes of this study, changes were made to the PD and typical curriculum for a unit on integer operations to promote teacher and student conceptual understanding through a process of mathematical discussion called argumentation. The teachers and students explored a comprehensive representation for integer operations called a vector number line model using the Texas Instruments TI-73 calculator Numln application. During PD, teachers engaged in argumentation to make claims about strategies to use to understand integer operations and to explain their understanding of how different representations are connected.

The results showed statistically significant growth in teacher PCK following the professional development and statistically significant growth in student understanding from pre- to posttest compared to the students who participated in the program the previous year. The findings also showed that there was a statistically significant association between teacher posttest PCK and student improvement in understanding even when controlling for years of teaching experience, teacher pretest knowledge, and student pretest score. This adds to the research base additional evidence that professional development focused on teacher pedagogical content knowledge can have a positive effect on student achievement, even with just a short period of PD (6 hours in this case).

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CHAPTER ONE: INTRODUCTION

Rationale

Improving student understanding of integers and operations with integers is important for many educators; however, there is not one instructional model for integer operations that seems satisfactory in explaining all integer operations. Integers are the set of positive whole numbers, negative whole numbers and zero. Integers are a number and operations concept that is foundational in mathematics, and used in applications in other fields such as science, business, and statistics. However, students struggle with this foundational concept, particularly around negative numbers (Ryan & Williams, 2007). A minor calculation error impacting the sign of a solution can have a serious impact on the answer because it may result in an inaccurate interpretation of the situation. For example, in a physics class, a solution to a problem related to force that is inaccurately calculated as a positive number results in a misunderstanding of the direction of the force. Many students struggle with understanding integers and accuracy of computation with integers. This weakness is seen in errors on student work in middle school math computation problems, high school algebra equations, and even in college mathematics, statistics, and science coursework.

The mathematics education community has struggled to find an instructional model that effectively supports students in constructing a rich conceptual understanding of integers and of negative numbers, in particular. Such a model would need to address the purpose of the negative numbers and justify the arithmetical operations on them in order to be comprehensive. A literature review of instructional strategies, curriculum, and research on integers revealed the following models commonly used in classrooms K-8: Annihilation model with counters or chips (Baroody & Coslick, 1998), movement on the number line (Baroody & Coslick, 1998; Cemen, 1993; Davidson, 1987, 1992; Page, 1964); transformations of an object's position and state (Schwarz, Kohn, & Resnick, 1993/1994; Thompson & Dreyfus, 1988); elevator or elevation model (Dahl, 1972; Froman, 1973; Janvier, 1983, 1985; Luth, 1967); use of metaphors for example, people getting on and off a bus (Davis, 1967; Streefland, 1996; Williams & Linchevski, 1997); technology, such as, calculators and virtual manipulatives (Browning & John, 1999; Utah State, 2008); and other real world applications, to name a few, money, temperature, and yardage in football (Davidson, 1992; Sheffield & Cruikshank, 2001).

The purpose of an instructional model is to add "obviousness" and "correctness" to mathematical concepts, but this purpose is not achieved by the current models used in most textbooks related to integer operations (Linchevski & Williams, 1999). Moreover, these models lack the comprehensiveness needed to address all operations with negative numbers (Fischbein, 1987, 1994). Due to the lack of a comprehensive instructional model, many students struggle with understanding integers and integer operations, developing misconceptions about these numbers (Ryan & Williams, 2007). This study is a step towards developing an effective and comprehensive instructional model using a number line vector representation along with professional development (PD) for teachers to build pedagogical content knowledge (PCK) focusing on engaging students in classroom discourse through argumentation. There is no clear evidence supporting one model, and there is a need for more research on representations for integer operations.

Pedagogical Content Knowledge

Teaching mathematics is a complex profession which requires more than just knowledge of the subject matter. According to Fennema and Franke (1992), the components of mathematics teachers' knowledge include the following: knowledge of mathematics, knowledge of mathematical representation, knowledge of students, knowledge of students' cognitions, and knowledge of teaching and decision making. Fenema and Franke (1992) emphasize the importance of mathematical representations in helping students connect abstract mathematics into something they can relate to and understand. Some researchers have focused on developing what is referred to as pedagogical content knowledge (PCK). According to Shulman (1995), PCK includes the following:

> The ways of representing and formulating the subject that makes it comprehensible to others . . . an understanding of what makes the learning of specific topics easy or difficult; the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (p.130)

Grouws and Schultz (1996) also include in their description of PCK the ways teachers provide students with useful unifying ideas, clarifying examples and counter examples, helpful analogies, important relationships, and connections among ideas which gets at more of the behaviors of teachers rather than just the kinds of knowledge they possess.

The theoretical framework for this study is informed by this research on the relationship between teacher PCK and student learning. Turnuklu and Yesildere (2007) in their research of PCK in mathematics of 45 pre-service teachers in Turkey and their approaches to teaching fractions, decimals, and integers, found that many of the pre-service teachers had difficulty understanding student misconceptions related to integer operations, facilitating mathematical discussions that would engage students in mathematical thinking about integers, and asking questions to assess student understanding. Their work was also informed by the research of An, Kulm, and Wu (2004) which they used to construct the following theory: (a) PCK is associated with teacher beliefs, content, knowledge and how one teaches, (b) Teaching is associated with PCK and knowledge about students' thinking, (c) Knowledge about students' thinking is associated with addressing students' misconceptions, engaging students in math learning, building on students' ideas, promoting students' thinking about mathematics, and (d) Student learning is most closely associated with knowledge about students but due to the interrelationship of the model, Student learning is associated with all of the four components. This integrated approach of improving teacher PCK, developing the ability of teachers to facilitate conversations about mathematics among students in their classrooms, and the resulting student conversations are proposed by this researcher as keys to improving student learning.

Proposed Intervention Based on Pilot Study Results

This study was informed by the findings of a pilot study this researcher conducted in May 2008 of 60 students from grades 7, 9, and 11, which revealed a diversity of strategies used to solve problems involving integer operations. The greatest area of inaccuracy and misconceptions was with subtraction of integers. Challenging such misconceptions requires an innovative instructional intervention; the use of a number line and a vector model as a representation for integer operations is a proposed strategy in need of additional research due to limited prior research in comparison to other representations. A vector number line model of subtraction of integers was used as a representation where subtraction is represented as a directed difference between two locations and multiplication is represented as repeated addition or repeated subtraction. Based on a review of literature on transformative PD, it was decided that argumentation would be used as an innovative instructional strategy to challenge teacher and student misconceptions around integer operations and to encourage teachers and students to share their mental models for integer operations.

Student Discourse about Mathematics

There is growing evidence that students in elementary, middle, and high school can reason, justify their thinking, make claims and warrants in a supportive classroom that is a mathematical community (e.g., Enyedy, 2003; Francisco & Maher, 2005; Goos, 2004; Maher, 2005; ; Maher & Martino, 1996; Mueller, 2007; Mueller & Maher, 2009; Yackel & Hanna, 2003). However, developing this kind of environment requires expectations for behavior, norms for making claims and warrants, activities that promote different ideas, and facilitation of the development of reasoning through teacher questions to advance student thinking (McCrone, 2005; Yackel & Cobb, 1996).

Most mathematics teachers are familiar with mathematical reasoning, but formal reasoning is often reserved for students in high school when they are taught how to construct proofs as mathematical arguments using accepted statements considered facts such as definitions and theorems established by the mathematical community. They learn the acceptable form of justifying their thinking in the proof format expected by their teacher or the textbook. However, Stylianides (2007) and Francisco and Maher (2005), argue that this concept of mathematical proof can occur as early as elementary school where students are given opportunities to reason and justify their thinking. Students and teachers can collaborate to establish classroom norms for what is acceptable for a proof in their class community. These are important prerequisites for future work with more formal mathematical proofs which are used by participants in the larger mathematics community to communicate to other mathematicians. Therefore, it was determined that mathematical reasoning using argumentation would be an age appropriate activity for grade 8 students in the Jumpstart program.

Teachers play a key role in setting up a safe environment for students to share their thinking with others and to argue their point of view in a public way that is productive (Yackel & Hanna, 2003). When teachers have low expectations of students in engaging in productive discussions, they deprive students the opportunity to learn from one another through the process of developing an argument, making a claim, and justifying their thinking (Mueller & Maher, 2009). The purpose of this study was to engage teachers in argumentation to challenge their understanding of integer operations and to have them experience the value of argumentation about doing mathematics during PD to develop their PCK. Then the PD would develop teachers' ability to facilitate classroom conversations about mathematics among their students in small groups and whole class discussions by creating, with students, expectations and norms for argumentation and selecting key activities that stimulate argumentation where students are motivated to share different strategies for solving expressions with integer operations and to justify their solution.

Methods and Research Questions

This research was conducted during a three-week summer intervention called *Jumpstart* for grade 8 students in a large urban school district in Central Texas. This program serves students who had not passed the state mathematics assessment, Texas Assessment of Knowledge and Skills (TAKS) test, required for promotion to high school. However, as part of the Student Success Initiative[1], students who pass grade 8 classes and attend Jumpstart, if approved by their grade placement committee, are promoted to high school despite their failing TAKS score. Each year, the Jumpstart program administrators are challenged to find teachers to teach in the program. Most of the teachers participate in summer school, so few remain that are available or interested in teaching Jumpstart. Each year there have been several first year teachers teaching in the program. There is a need to ensure that the teachers understand the mathematics in the Jumpstart program as well as understand common misconceptions that students may have by including mathematical content and pedagogy knowledge features in the PD for the Jumpstart program.

This research study was considered an intervention that supplemented the current PD that has occurred in the past two years with additional emphasis on the development

[1] Enacted by the 76th Texas Legislature in 1999 and modified by the 81st Texas Legislature in 2009 to ensure that all students receive the instruction and support they need to be academically successful in reading and mathematics.

of teacher PCK related to integers. The program included 22 teachers and 341 students. Teachers attended two days of PD prior to the start of the program. The first day of PD constituted the intervention for this study. It focused on improving teacher understanding of integers and integer operations. The second day of PD focused on the algebraic patterns lessons and activities that are not a focus of this study.

During the PD, the teachers engaged in activities to explore how to build on students' ideas during mathematical discussions in the classroom and how to set up class structures and activities to promote small group discourse about mathematics using argumentation. The PD activities focused on motivating and encouraging teachers to share their thinking and talk about the mathematics they use to solve problems. Through these experiences they were challenged to support their own claims about their understanding of integers which deepened their understanding and enabled them to experience the value of argumentation. This study was conducted to determine if PD for teachers using a comprehensive vector model for integers and using argumentation to promote the sharing of different solutions, diverse strategies for problem solving, and to challenge misconceptions would improve teacher PCK. This study will also show whether there is a relationship between teacher PCK and student understanding of integer operations.

Data were gathered to explore the effect of the PD on teacher PCK through a pre and post assessment. Student achievement data was collected using pre and posttest results to determine whether teacher PCK is associated with changes in student achievement. The *Constructivist Learning Environment Survey* (CLES) created by

Taylor, Fraser and White (1994) was used to gather evidence of the existence of the kinds of classroom practices related to student discourse and argumentation emphasized in the PD.

The hypothesis was that teacher PD can potentially affect teacher PCK and potentially student understanding in the following ways:

1. Exposing teachers to activities that apply integers in real world contexts, such as vector forces and temperature change and representing integer operations with a comprehensive model using a number line will improve teacher understanding of integer operations and their ability to connect the rules for operations to a relevant context.
2. Providing teachers with experience in argumentation around misconceptions and solution strategies, and opportunities to discuss and create structures to use in the classroom to facilitate argumentation will enable teachers to facilitate student argumentation around misconceptions in their classroom.
3. Students who are provided with opportunities to engage in classroom discourse and argumentation about doing mathematics will improve their understanding of integer operations and perform better on an assessment of that understanding than they had prior to engaging in these activities.

These three hypotheses are shown in the proposed Theory of Change in Figure 1.

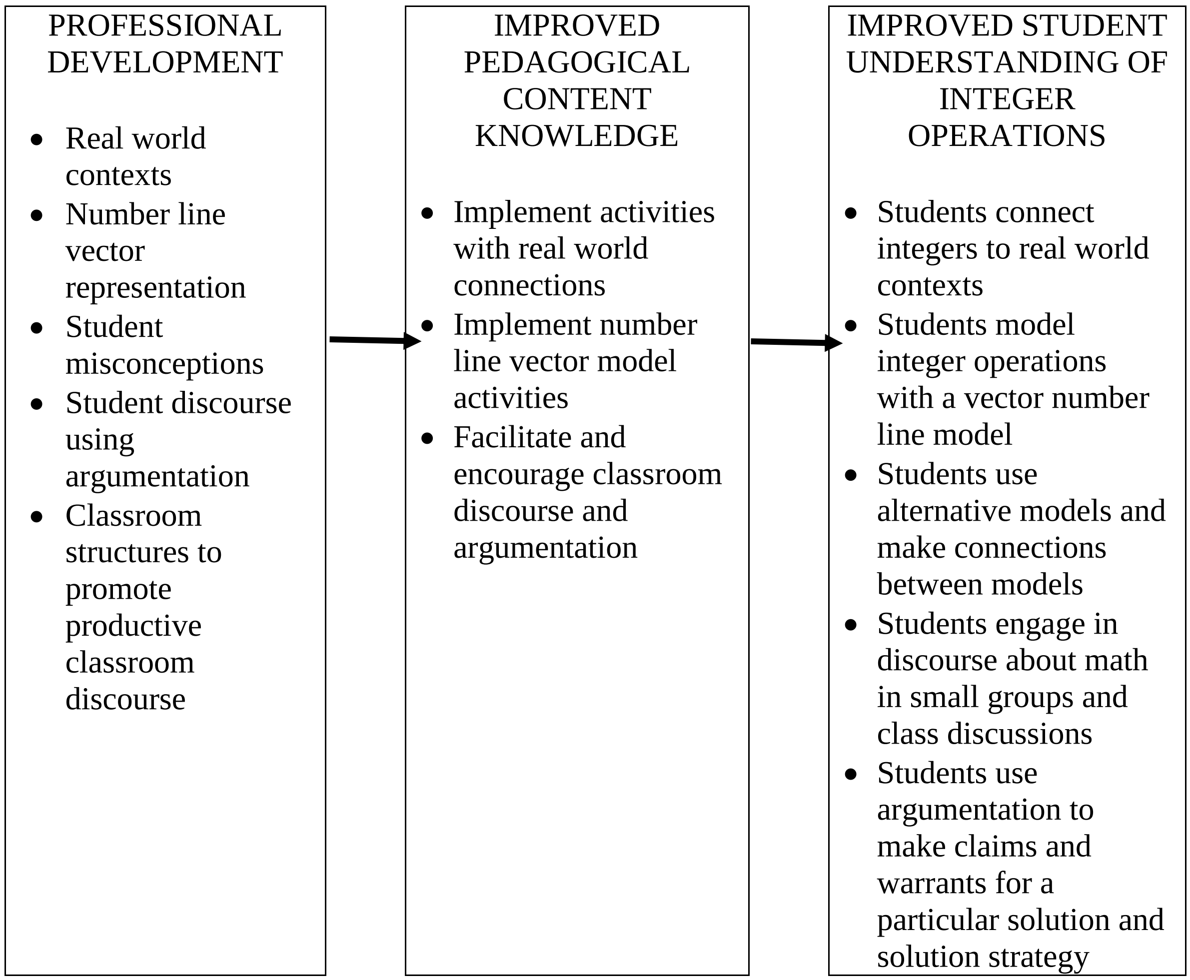


Figure 1   Theory of change for this research study created by Sarah Harris 1/15/10

In order to examine the impact of PD on teacher PCK, changes between a pre- and posttest of teacher PCK that related to the PD content and the curriculum the teachers implemented was analyzed for statistical significance. In order to explore any differences in student understanding based on teacher change in PCK, changes between a pre and post student assessment were compared to the changes between pre and post student assessment for a comparison group of students who attended Jumpstart 2009.   A hierarchical linear model was used to examine the hypothesis that there is a relationship between the strength of a teacher's PCK and student understanding. The results of teacher responses to the CLES survey were analyzed with descriptive statistics to better understand the extent to which they provided students with opportunities to engage in discourse practices emphasized in the PD and their responses were compared to the responses of their students to determine whether students experienced similar opportunities to engage in classroom discourse as their teachers described. Focus group interviews of Jumpstart students were conducted by a grad student volunteer to learn more about changes in student understanding over time and the impact of the instruction and activities on their understanding.

Specifically, the five research questions were as follows:

1. What are the general patterns of teacher content and pedagogical knowledge of integers based on responses to a pre and post assessment?
2. To what extent does PD impact teacher content and pedagogical knowledge as measured by growth between pre and post assessment?
3. Is there a statistically significant difference between the growth between students' pretest and posttest scores for Jumpstart 2010 compared to the growth made by students in Jumpstart 2009?
4. Do differences in teacher content and pedagogical knowledge explain more of the variance in student performance (pretest/posttest) than years of teaching experience of a teacher?

5. What is the relationship between teacher responses to the Constructivist Learning Environment Survey (CLES) and the student responses to the CLES?

A Hierarchical Linear Model (HLM) was used to examine the relationship between teacher content and pedagogical knowledge of integers and mathematics learning related to the fourth research question. A two-level Hierarchical Linear Model with student math scores as the level one outcome variable (a within-class model) and individual class variables, such as mean math score, as the level two outcome variables (a between-class model) was used to determine the variance in student test scores from pre to post-test as a function of their class assignment. Classroom-level predictors (teacher content and pedagogical knowledge of integers and years of teaching experience) were added into the level two models to determine whether they explained the variance in mean class scores and pretest/posttest performance slopes. Because the relationships between variables measuring student learning are at least partially dependent on the larger context (the classroom) in which learning occurs, teacher PCK should partially explain the variance across classes, as should the years of teaching experience of a teacher. It was of interest to see whether PCK following PD would outweigh experience in explaining the variance across classes.

What follows is a roadmap to the rest of this report. In chapter 2 three research reviews are provided: first, the research related to instructional strategies for improving children’s understanding of integers in K-12; second, research on classroom discourse among students about mathematics, focusing on use of argumentation about mathematics;

and finally, the research that informed the creation of the teacher PD to assist teachers in teaching students about integer operations as well as the theoretical background for considering how PD might impact student learning. In chapter 3 a description is provided of a pilot study this researcher conducted and how it informed this dissertation research project. The methodological approaches used to analyze data and draw conclusions as well as a background of the study and its participants are provided in chapter 4. In chapter 5, the findings of this study are presented. Information from student focus group interviews is presented in chapter 6 to add further background information for the context of this study. A conclusion, limitations, and recommendations from this study are provided in chapter 7.

CHAPTER TWO: LITERATURE REVIEW

To inform the design of this research study, three lines of research were reviewed: how to develop understanding of integer operations through instructional strategies and representations; how to use mathematical discussions to improve understanding, specifically one method called argumentation; and how to develop and research teacher PD to improve teacher PCK and eventually improving student understanding. Based on this literature review, this researcher suggests that there is a need for additional research on the topic of integer operations focusing on instructional strategies to improve student understanding and professional development to improve teacher understanding, which is the focus of this dissertation research study.

Developing an Understanding of Integer Operations

To determine what background the Jumpstart teachers and students may already have related to integer operations and what research has shown to be most effective in terms of instructional strategies and representations, a review of literature was conducted focusing on three main developmental time periods in a child's education: early elementary, middle grades, and high school. This first review of literature, shows the work that has been done, primarily with small samples of students, and mostly of a qualitative and descriptive nature, to begin to understand instruction related to this mathematical concept of integer operations. This researcher suggests that what are needed to move forward are larger scale studies, with qualitative and quantitative methods, to begin to inform future curriculum development and instruction for pre-service teachers and in-service teacher professional development.

Children are typically first introduced to negative integers in upper elementary grades, using two informal semantic models. These models are helpful because they are direct examples of the properties of integers (Schwarz, Kohn, & Resnick, 1993, 1994). In the *annihilation* semantic model, children are told that there are positive integers and negative integers, that they are represented as collections of positively charged and negatively charged particles (respectively), and that there is a rule that allows one to cancel out a positive particle by a negative particle resulting in what some call a "zero pair" (Hayes & Stacey, 1999; Liebeck, 1990). In the *extended number line* semantic model, children are told that the natural number line they have been using extends not only to the right of zero, but it also extends to the left of zero. With this model they learn that positive integers correspond to movements to the right, and that negative integers correspond to movements to the left (Hativa & Cohen, 1995; Kent, 2000; Moreno & Mayer, 1999; Thompson & Dreyfus, 1988; Varma, Harris, Schwartz, & Martin, 2009).

The annihilation and extended number line models allow children to solve integer addition problems and some integer subtraction problems; however, they are not as good at explaining the procedure for subtracting a negative integer, because situations require the addition of zero pairs in order to physically subtract from a given amount (Hayes & Stacey, 1999; Liebeck, 1990; Moreno & Mayer, 1999). For this reason, many teachers transition their students quickly to a more symbolic understanding of integers once they have the basic intuitive understanding mastered (Thompson & Dreyfus, 1988). In middle school, students are often taught rules for integer arithmetic, such as that rule that says when a negative number is multiplied by a negative number the result is a positive answer. Then in high school, students are introduced to an additional rule for multiplying an inequality by a negative.

Based on the researcher's review of literature on students' developing integer understanding in grades K-12, it appears that the mathematical community has been in search of a *model* that will satisfy the need for the negative numbers, justify the arithmetical operations on them, and explain the relationship between the operations. Fischbein (1987) argued against using the existing models for negative numbers (annihilation and number line models). Fischbein explained that the models lack "comprehensiveness," and were based on artificial conventions which do not address the challenges that students are faced with when working with negative numbers. This literature review will lay the foundation of the current practices in K-12 education related to integer understanding as well as to the research on strategies for improving such understanding in order to illuminate the current state of what is known about the development of integer understanding in students and to inform the development of the research questions for this study.

Laying a Foundation: Kindergarten to Grade 2

Number Concepts

Before introducing integer concepts to students in upper elementary grades, it is important to consider the foundations that have been laid in grades K-2 in order to connect with prior learning about numbers and operations with whole numbers. An understanding of ordinal and cardinal properties of numbers is also an important prerequisite for students in Kindergarten to understand integers. In a study conducted by

Davidson (1992) of 4-7 year olds Davidson found that children use actions that are familiar to them to represent negative quantities. At this age, the children most often described zero as “nothing” instead of an actual number. At these ages, the children also showed the beginning ability to combine positive and negative amounts. The findings showed that students had difficulty coordinating the cardinal and ordinal[2] meaning of negative numbers, which Davidson attributed to the lack of experience in the children’s everyday lives with ordinal evidence compared to cardinal evidence of negative numbers.

Davidson suggests that the following activities be used with students to develop intuitions about ordinal properties and their relation to cardinal properties: (a) moving steps from a starting point and then retracing the steps back to and beyond the starting point, (b) marking the path with units to make it easier to retrace one’s steps, (c) verifying the number of steps required to arrive at the new position and then confirming when retraced that the same number of steps were completed, and (d) using the concept of a starting point as an intuitive basis of zero (p. 21).

Page (1964) showed how the number line can be explored by young students with a cricket jumping along the number line. Page shares a few ways that young children spontaneously discuss numbers that are negative: “under two,” “goodbye two,” “two below zero.” Some students even come up with unique ways to label the numbers below the zero such as B1, B2, B3, for below by 1, below by 2, etc. (Baroody & Coslick, 1998, p.8.22).

[2] Cardinal properties relate to the size of a set of numbers (i.e., set A {1, 2, 3, 4, 5} has a size of 5) and ordinal properties relate to the position of a number (i.e., 1st, 2nd, 3rd).

Representations

There are different representations of numbers that students explore at this early age. Using concrete objects, such as counters, to represent and develop number sense is one recommended way to introduce counting to young students (National Council of Teachers of Mathematics, 2000). The concept of zero can be introduced by putting different amounts of counters in different boxes and leaving one box empty to represent zero (Sheffield & Cruikshank, 2001). When exploring real world situations, students may naturally discover the negative numbers or ask questions about such things, which will be a way to get them thinking about integers at an early age. For example, one might show students the negative numbers on a thermometer and discuss what kind of temperature would be measured in negative degrees (Sheffield & Cruikshank, 2001).

As these studies have shown, there are valuable experiences that children in K-2nd grade can be exposed to in order to begin learning about integers.

Early Introduction and Exploration: Grades 3-5

Typically students begin to be introduced to and to explore integers (positive and negative numbers) in grades 3 through 5. This is often done through activities which extend the number line to the left past zero, exploring real world situations of temperature or bank accounts, where negative numbers are needed, and through literature connections that introduce the larger world of numbers to upper elementary students. However, one group of researchers studied the effect of early instruction in the rules of integer operations with third grade students and found that the students' knowledge of rules for operations that they were taught in school negatively effected their problem solving

(Nunes, Schliemann & Carraher, 1993) as they attempted to apply these rules without any conceptual understanding. For the most part research for this age group has focused more on conceptual development through different models or representations of integers and integer operations (Cemen, 1993; Hativa and Cohen, 1995).

Number Line Activities

Cemen (1993) in an article in "The Arithmetic Teacher" described a strategy for introducing integer concepts to young students with a number line. A large number line could be made on brown paper or computer printer paper. It should be large enough for all to see when it is taped to the board in the class room. Students would be given their own small number line for their desk that could be used with a cut out figure of a person to move around along the number line. Students then could discover the computational rules. For example, adding two numbers with the same sign would mean moving in the same direction on the number line. When the signs are different, the figure would be moved in the opposite directions. Cemen explained that when the numbers have different signs, they "pull" against each other as in a game of tug-of-war. The sign of the sum would be the sign of the one that pulls harder. Subtraction on the number line would be illustrated by turning it around, which distinguishes between the negative sign as part of the number and the subtraction sign as an operation. Subtracting a negative would mean turning the figure around and walking the figure backward. This would be the same as adding a positive (not turning around and walking forward). This eventually would lead to the rule of adding the inverse. This strategy gets at concepts of magnitude (size of movement), direction (changes in location), and inverse.

Hativa and Cohen (1995) conducted research in two fourth grade classes in Tel-Aviv, one was assigned as treatment and one as control. They used a computer program called *The Challenger* that provided a number line model for students to use to explore integer operations to reach a target number. Hativa and Cohen (1995) classified five types of problems and related student misconceptions that involved integers: (a) subtracting a positive number from zero, (b) subtracting a positive from a smaller positive number, (c) adding two negative numbers, (d) adding opposites ($^{-}7 + 7$) , and (e) adding a positive to a negative. They found that when students are given a problem involving subtracting a positive from a smaller positive number (4 – 7) that students often would solve it the same as they would the reverse (7 – 4) as if the commutative property held true for subtraction. When given a problem involving adding two negative numbers, students often subtracted the numbers, because they assumed that a minus sign anywhere meant subtraction. When given addition of opposites ($^{-}7 + 7$) the students would often just add the magnitudes (14). These challenges involve an undertanding of operations and signs that are very different mathematical concepts. Students who were in the treatment group made statistically significantly greater gains in achievement based on pre and post treatment interviews.

The research of Cemen (1993) and Hativa and Cohen (1995) informed the development of the PD and the curriculum for this study. The teacher PCK assessment for this study included two items from the research of Hativa and Cohen (1995) to assess teacher undetstanding of the difference between an operation and a sign in an expression.

The research of Cemen (1993) informed the development of teacher and student activities using the number line representation for this study.

Investigations: Grades 5-8

Number Line Model

Several researchers have studied the use of number line models with children using computer worlds for their investigation of integer operations. Thompson and Dreyfus (1988) conducted a small study observing two grade 6 students over a six week period as they used a computer program to move a turtle on the screen right and left to explore integer operations as transformations. A negative number command would cause the turtle to turn around and walk the number of steps. There is a distinction made between the state (position) and the integer (change of position). After the turtle completed its movement it would turn back around. The students were asked to predict the result of a command, execute the command, and then discuss their prediction. The goal was for the students to move towards forming a generalization. Students learned that a "negative - negative" is turning around twice. To solve the expression 3 – (-5), the turtle would walk three steps forward, turn (subtract), and then turn again (negative) to face the same direction before walking five steps forward.

Schwarz et al. (1993, 1994) conducted a similar study with trains in a computer environment called "Trainworlds" with 4 grade 5 students. The trains in this program had magnitudes and were color coded (white, grey) for direction. The operations of cutting or gluing trains were used to represent binary addition and subtraction. Loading and unloading trains were used as unary actions (positive or negative) on trains.

Comparing trains was used to demonstrate relations between negative numbers. Three of the four students were able to successfully map the situation in the posttest to the train model to solve the problem.

Colthorp (1968) also researched use of the number line model by conducting a study with two teachers each teaching one intervention class and one control class of grade 6 students. The intervention class was given instruction with a concrete number line approach and the control class was given more of a rule based instruction using the algebraic approach. There were no statistically signficicant differences in achievement at the end of the program between the groups.

In summary, the number liine model shows promise for helping teachers and students understand conceptually the role of an operation and a sign in terms of the magnitude and direction of movement. However the results of the use of this representation on student achievement is mixed; therefore, additional research is needed.

Cancellation Models

Some might wonder which model or strategy is best, the counters or the number line? Van De Walle and Lovin (2006) suggest that students should be allowed to experience both models and explain how the two models are alike. They recommend developing an integer understanding using both models at the same time to help students connect the two models.

Wilkins (1996) conducted clinical interviews with 16 grade 6 students and identified the strategies students preferred to use to solve integer addition and subtraction problems and the accuracy of use of the different models. When given choice of a model to use, 50% of the students chose to use a mental model, 23% used a number line, and 27% used two-colored counters. However the students who used mental models were only successful 65% of the time, students who used the number line were accurate 70% of the time, and students who used colored counters were correct 75% of the time. The results showed that students who used the number line and two-colored counters were more successful[3] than those who used mental models. The problems students were given were situated within real world contexts such as temperature, weather, mail delivery. Students tended to use continuous models for continuous situations (number line) and discrete models for discrete situations (counters).

Ball and Hill (2009) observed a grade 7 mathematics teacher conducting a lesson on integer operations with black and red chips. The students would match up each black chip with each red chip and then count the chips left over. However when the teacher modeled how to do a subtraction problem with these colored chips, the students had difficulty understanding the representation. The teacher observed was also not sure of the representation and decided to put the colored chips away and focus on a rule or strategy for converting the subtraction into addition, but then confused the students by trying to connect to a real world experience of money and debt. In order to have continued with the chips, Ball and Hill (2009) point out that the teacher could have referenced the usual "take away" or "regrouping" method, by thinking that three red chips need to be taken away, but there are only two to take away, so another red chip can be created by adding on a zero pair (one black chip and one red chip). Once three red chips are removed, there

[3] Due to the small sample size of this study, significance tests were not conducted. Only descriptive statistics were provided.

would be two black chips left. This shows that chips can be a challenging representation for modeling subtraction.

Hackbarth (2000) conducted research with of 68 grade 7 students and looked at the impact of two diferent manipulatives versus instruction just using the rules for integer addition and subtraction. One group (N = 23) was given “plus-minus” pieces, another group was given two colored chips (N = 22) and the control group (N =23) was given instruction on the rules for the operations. The plus and minus pieces were created from tile spacers in the shape of a plus and ones that had both sides cut off to form a minus. The colored chips were red on one side and yellow on the other side. The rule was based on the following abstract method $a - b = a + (-b)$. Hackbarth (2000) found no statistically significant difference between the performance on pre, post, and rention tests (teacher made) of the three groups based on instructional method. This suggests that cancellation may not be the best instructional approach.

McCorkle (2001) compared two groups of grade 7 students, based on performance on a pretest, posttest and follow up retention measure on integer undertanding of operations of addition and subtraction. The intervention group received instruction using a relational approach where students were able to manipulate a thermometer scale with the concept of “hotcubes” and “cold cubes.” The control group was taught with an instrumental textbook approach using the algebraic rules. Students in the intervention group on average scored higher and had better retention than students in the control group. The results of this study suggest that use of cancellation may be associated with student understanding of addition and subtraction of integers

In summary, the effects of cancellation models are mixed. McCorkle (2001) found positive results of using cancellation with addition and subtraction. Hackbarth (200) found no significant difference between achievement of students following instruction with rules compared to use of cancellation. Ball and Hill (2009) found evidence of cancellation being challenging with subtraction for both teachers to implement and for students to use as a representation. Therefore, the evidence does not show that use of counters and a cancellation model is more effective than the number line representation.

Rules

To determine whether rule based understanding is better than conceptual understanding based on physical models, Harvey and Cunningham (1980) conducted research by assessing integer understanding of 163 grade 8 students with a 46 question assessment. The students who were most successful at integer addition used a physical model such as the number line to help them understand the problems they were asked to solve. Subtraction of integers was the most challenging for students, and only students who converted subtraction to adding the opposite of the subtrahend (second number) were successful in solving the subtraction problems. Therefore, this rule was more effective for subtraction.

Real World Contexts

There are many real world contexts for the practical applications of integers. Janvier (1985) developed a model based on the work of Luth (1967) that uses a concept of elevation of a hot air balloon basket attached with changing amounts of balloons (+)

and sand bags (-). This model works well for addition and subtraction, but does not provide students with an opportunity to work with multiplication and division.

Davis (1967) came up with a creative story to help students understand integers with the context of a postman delivering mail daily to a woman named "Mrs. Housewife." Sometimes he delivers her checks (positive numbers) and sometimes he delivers her bills (negative numbers). Each day she figures out how much money she has or owes. According to Davis (1967), this context worked well to help students understand subtraction of integers, because subtraction is when the postman returns to take away the wrong mail (subtracting a negative number would mean taking away a bill).

Metaphors and Stories

Williams and Linchevski (1999) studied student's "situated intuition." Situations and models describe a reality that is meaningful to students, in which the extended world of negative numbers already exists and the students' activities allow them to discover it. Williams and Linchevski designed two experiments to give the students a concrete situation for exploring the integer concept by using an abacus. They created two real world simulations that could be enacted using two colors of beads on a double abacus. In the first game, *Disco Dance*, Williams and Linchevski used a double abacus with students to explore addition and subtraction in order to allow for an extension of the children's existing number schemes. They situated the double abacus activity within the context of a disco game where dancers were arriving or leaving through a gate. The children had to keep track of the number of dancers with the double abacus after drawing a card that would tell how many dancers came or left. This involved cancellation and then compensation if they ran out of beads. The purpose was to explore situated intuition using children's everyday common sense and intuition in the disco problem scenario. There was an intuitive gap where subtraction was introduced, which caused the researchers to believe that this might not be the best situation for exploring integer operations. Streefland (1996) explored a similar context with a "Bus Stop" scenario with people getting on and off of a bus. This context may have been more familiar to some students than the disco dance, and students have used colored chips more easily than beads to model the activity.

Williams and Linchevski (1999) designed a second game with dice since the disco game was not as intuitive for subtraction. In order to make subtraction more concrete, they provided students with blue (+) and yellow (-) dice to represent positive and negative. Then they introduced a third dice with all faces labeled either "add" or "sub" for subtract. The children were told to first throw the blue and then yellow dice and determine the resulting score. Then they would throw the add/sub dice and either add or subtract their new score to their previous total. So if their previous total was a $(^{+}2)$ and they rolled a $(^{-}3)$ and a $(^{+}7)$, combined the new score would be a $(^{+}4)$. If they rolled "sub," then they would subtract their new score $(^{+}4)$ from their previous total $(^{+}2)$ which would result in $(^{-}2)$. The students worked in teams of two, each student represented by a bead color. However, a weakness of this game was the arbitrary assignment of plus and minus symbols to the teams that led to some new difficulties which had to be resolved through discussion of fairness in the game situation. The researchers determined neither the disco game nor the dice game was able to be extended to include multiplication and division.

Chiu (2001) engaged a group of 12 middle school students and 12 post-secondary adults in an exploration of integers within the context of the stock market. Students were then interviewed and asked to explain the metaphors they had developed. This involved the explanation of six arithmetic expressions. When solving problems, children used metaphors (M = 3.67 times, SD = 2.99) twice as often as the adults (M = 0.25 times, SD = 0.62) and the difference was statistically significant (t = 2.97, p < .02). When showing understanding of the six arithmetic expressions, adults used metaphors more often (M = 7.00 times, SD = 2.70) than children (M = 2.25 times, SD = 0.35), which was also statistically significant (t = 3.40, p < .01). Children were more accurate when they used a metaphor (93%) during calculation, than when they used other methods (81%, χ^2 (1, N = 331) = 5.48, p < .02). However, children were slower when using a metaphor (M = 5.93 seconds, SD = 2.73) than when they used other methods (M =3.14 seconds, SD = 1.27, z = 6.28, p < .001). Chiu (2001) found the use of three main metaphors: use of objects (e.g., drawing bars that cancel each other out), motion (e.g., talking about location and walking a direction), and social transactions. (e.g., describing owing or borrowing money). The authors concluded that no matter how effective metaphorical reasoning is, it is slower than facts or algorithms.

Games

Baroody and Coslick (1998) share a golf game created by a teacher named Mrs. Kail for integer addition. A group of four students are given a paper bag containing red and green dice with sides labeled from 0 to 5. The students play a 9-hole golf game. For each hole, they take turns reaching into the bag to pull out a die. The color and the number determine the person's score. If it is a green 4, then they are 4 over par. If it is a red 4, then they are 4 under par. For each new hole the players' new score is added to their old score. The player with the lowest score wins. In the beginning, the students had difficulty determining their score, so the teacher provided each group with a pile of green and red chips to use to model the score changes and check their calculation. The students had an easier time coming to an agreement about their new scores by having the chips to justify their line of thinking.

Students in grades 5-8 are often introduced to integers for a specific unit or short period of time, much less than the time that they are exposed to positive whole numbers. Baroody and Coslick (1998) suggest that teachers have a classroom economy system where students earn a paycheck each week. They can spend their pay on items in a class store or minutes in an educational entertainment center. They can lose money by misconduct, incomplete homework or assignments, or by other means. They can gain money by collaborating with others, peer tutoring and making good effort in class. Students can be the ones playing the roles of banker, store cashier, etc. to calculate transactions and balances in a ledger book for each student (Faulk, 2007). However, no evidence was provided about any association between use of this kind of classroom economy system and student performance on integer operation problems.

Instructional Technology

To determine whether instructional technology could be used to improve student understanding of integers, Anctil (2002) researched whether students who used computer-assisted instruction with a program *Walking on the Number Line* in a middle school (grades 6-8) improved in their understanding of addition and subtraction of integers. The *Walking on the Number Line* program was created with Director 8.0 softward on a Macintosh computer. Anctil administered a similar pre and post test to students with ten addition and ten subtraction of integer problems following the computer lessons. There was a statistically significant difference between pre and post-test performance (t = 2.54, p = .017). However, this study included only thirty student participants, twenty improved from pre to postttest, but ten either stayed the same or decreased their score on the posttest.

Kalu (2006) designed a computer assisted instruction (CAI) program entitled *Working with Integers: Addition and Subtraction* and field tested it with a group of 32 urban grade 7 pre-algebra students with a history of low motivation, poor attendance, and low achievement. The computer program was created with *Macromedia Director* MX (Educational Version). The computer program contained three sections: concepts and properties of integers, integer rules, evaluation and posttest. The concepts and properties section exposed students to three methods of solving integer problems: calculation, absolute value method[4], and the number line method. The students used the program for

[4] This is a rule where the first step is to add the absolute values (or magnitudes) of each number and then the second step is to keep the sign of the number with the largest magnitude if the signs are different and to keep the original sign of the numbers in the problem if there signs were the same.

five days. The results showed significant improvement in the comprehension of concepts of adding and subtracting integers (t = 35.48, p < .01) showing that the computer assisted instruction program was effective in improving student understanding.

Remediation and Application: Grades 9-12

By high school students are expected to have learned about integer operations, since they are a prerequisite to Algebra 1. Any further instruction in high school on this topic would either be in the form of remediation or application to high school mathematics content. According to Bruno and Martinon (1999), the main reason students experience confusion while learning about the number system is that throughout their education, there is not a single unifying idea about numbers or perspective on the best way to teach students about numbers. They end up learning each part of the system in an isolated way (i.e., whole numbers, fractions, decimals, integers, irrationals, complex numbers, etc.). Bruno and Martinon conducted a research study of 12-13 year old students in an urban classroom in Spain. They explored three representations of integers: abstract, contextual, and the number line. They found that the number line, which most students did not initially know about, became an essential tool in the students' understanding of integers. Students obtained successful results for activities that relate the number line to concrete situations, when the number line was employed as an aid by students in solving problems (Bruno & Martinon, 1999, p. 808). Taking this number line representation from the middle grades and then extending it throughout high school to include graphing rational and irrational numbers is recommended by the National Council of Teachers of Mathematics (NCTM, 2000) in addition to instruction in new concepts to strengthen students' understanding of numbers such as vectors, which simultaneously represent magnitude and direction.

Moses, Kamii, Swap, and Howard (1989) researched urban students' understanding of integer operations through the Algebra Project where students learned about integers through a familiar context using a simulation of riding a subway. The students were introduced to the concepts of direction, displacement, and equivalence. Subtraction was modeled as comparing the endpoints of a pair of displacements. Then students worked within a coordinate system where they explored displacements with magnitude and direction. Then they connected this understanding in the context of the subway rides to integer addition. Students showed improved understanding of integer operations using the context of the subway rides and then moved on to more symbolic abstractions of the same concepts.

Abstraction with Variables and Symbols

When students begin studying Algebra, they are introduced to variables used with numbers in expressions and they are asked to simplify expressions by combining like terms. At this point, some students struggle with knowing which numbers to subtract and which numbers to add. Vlassis (2004) found that grade 8 Algebra students utilized several procedures to simplify the expressions. Some first used brackets to group the numbers and then group the common variables. Then they applied a sign rule that if there was a minus next to a minus it would result in a plus. Some of the students worked right to left assigning the minus signs to the wrong numbers. For example, in the expression 4 – 6n – 4n, a student would respond that the answer is 2n – 4. Others used the sign following the number. For example, $6y - 20 + 3y$, they would subtract the 6y and the 3y to get a result of 3y. Others used rules such as, $7 - 9 = {}^{-}2$ because 9 is the larger number. Some students referred to a number line concept as a "scale" and spoke of moving up and down the scale. Most of the students referred to the minus in between two numbers as subtraction. Some students saw the minus sign as a divider. No students were able to see that a minus could have two roles. According to Vlassis (2004), understanding the dual roles of a minus is important when simplifying expressions in Algebra. Vlassis concluded that the students' lack of flexibility with the role of the minus sign may be related to the traditional instruction they received with a focus on rules and less on conceptual understanding. As students transition from arithmetic to algebra, there are negative numbers that are constants, coefficients, or solutions. So it is important that they continue to understand the role and meaning of the minus and the negative in each context.

Science Application

In Physics, the distinction between whole numbers and integers is paralleled by the distinction between scalar and vector quantities. Mass and time are scalar quantities that have only magnitude, but not any particular direction. Weight is a vector quantity, because it has both magnitude and direction (Baroody & Coslick, 1998). Physics is not the only area of science with integer applications. A charged-particle box is an analogy that can be used to help students review integer operations in high school by their math and science teachers. This analogy could be appropriately intuitive because it builds on a student's familiar "add-to" interpretation of addition and change "take-away" meaning of subtraction (Baroody & Coslick, 1998). Any model of integer addition and subtraction should clearly distinguish between + and – signs indicating a direction and those indicating an operation (Baroody & Coslick, 1998). The charged particle metaphor meets the essential criterion. In the expression (+2) + (-3) = -1, the positive sign in +2 represents the positive charge (the direction of the charge) and plus sign between +2 and -3 represents adding charges to the box (the operation of addition). In the expression (-3) – (-2) = (-1), the negative sign in -3, -2, and -1 each represent a negative charge (the direction of the charge) and the minus sign between -3 and -2 represents taking away charges from the box—the operation of subtraction (Baroody & Coslick, 1998). This particle box concept can then be related to other concepts in science that involve the application of integer operations such as boiling and freezing points, chemical bonds, electric charges, magnetism, enthalpy (thermodynamics), acceleration/deceleration, and force vectors to name a few.

Limitations and Future Directions

Based on this review of the literature, it appears that the two most common models for integer operations seem to be the colored counters and the number line representations. There are a few disadvantages to the model with cancellation using colored counters. It is best used with smaller numbers, because it becomes cumbersome dealing with a multitude of conters. It is difficult to model multiplication and division of integers problems. Also it is impossible to extend the model to include rational numbers. There are also disadvantages to the number line model. Despite its comprehensiveness in being able to model all operations, there is disagreement about the best way to model subtraction on the number line. Some students are confused because the "take away" model is what they associate with subtraction more than a measurement model. Also some students have difficulty physically counting on a numberline, unsure if they should count the numbers or the spaces in between the numbers (Carr and Katterns, 1984). A larger issue is that students need to understand the concept of an integer before they begin modeling operations with integers which can confound the challenges with each of these models.

According to the models and modeling perspective of learning, "students go beyond isolated pieces of information to construct well-organized systems of knowledge, and they go beyond thinking within isolated topic areas to also integrate and differentiate ideas between topic areas, subject matter areas, or domains of experience" (Lesh, Lester, & Hjalmarson, 2003). However, due to the many ways students can be introduced to integers over their K-12 experience, they need to have a way to make sense of these different models and representations of integers in an organized manner, otherwise, misconceptions and confusion can develop. It is important for teachers to allow students to talk about their understanding and work with each other to clear up misconceptions (Ryan & Williams, 2007). What follows is a discussion of the research behind use of argumentation to improve student understanding in mathematics classrooms.

Use of Argumentation to Improve Student Understanding

According to Partnership for 21st Century Skills (2008), the 21st century skills that students need to develop to be prepared for careers in science, mathematics and technology require "thinking criticially; making judgments; solving complex, multidisciplinary, open-ended problems; and communicating and collaborating" (p.10). Cobb, Stephan, McClain, and Gravemeijer (2001) conducted a teaching experiment in a grade 1 classroom focusing on linear measurement. They involved students in the following activities: explaining and justifying their solutions, agreeing or disagreeing with the solutions of others, and asking questions to make sense of the explanation of others. The students established classroom norms for acceptable solutions and explanations. Cobb et al.'s research lays a framework for research methodology to further study argumentation in the classroom. Argumentation is proposed as an innovative approach to address the challenges around changing student misconceptions and the need for students to develop more connected understanding of the procedures they use.

Enyedy (2003) conducted a study of two grade 7 math classes in an urban middle school. The participating students were ethnically diverse, and a majority of the students were from low-income homes. Students used computers with a *Probability Inquiry Environment* (PIE) interface that made agreement and disagreements between students public so that the argument could be solved. The interface included an agreement bar that could be moved to represent the extent to which one agreed with the response of another. When a student lowered the bar to show disagreement, a conversation would result as students argued their perspective to negotiate to a point of agreement.

Enyedy (2003) structured student predictions of solutions according to Toulmin's (1958) work with argumentation where productive argumentation occurs when a student engages in an argument by making a claim and providing a warrant. These activities

Enyedy refers to as “distributed,” because it takes interactions among multiple students to accomplish the activity. They are also “emergent” in that the activity can change during the interactions among students (Enyedy, 2003). However, this can only happen when students engage in this kind of activity undisturbed. Teachers can be what Enyedy calls a “limiting factor,” because they can provide constraints that limit this kind of activity to only certain times or types of activities (p.366). This limitation prevents the development of reasoning skills and use of mathematical tools. Enyedy followed two students and the changes in their reasoning during activity with the computer interface, then through a transformation of understanding following interaction with peers, and finally through a whole class discussion. The students who participated in the PIE classroom outperformed students in a comparison classroom based on statistically significant pre and posttest performance gains, $t(97) = 3.4$, $p < .001$ (Enyedy, 2003, p. 395).

Mueller and Maher (2009) conducted a research studying of 24 grade 6 students who participated in an informal afterschool program working collaboratively to solve problems. During the problem solving sessions the students were observed constructing arguments, justifying their solutions and solution strategies. These informal arguments led to more of a proof format. What developed was a mathematical community as defined by Goos (2004) as a place of inquiry where students can have a place to work on problems and share their mathematical thinking by proposing and defending arguments, responding to the claims of others tudents, and sharing their ideas. Students were observed solidifying their own understanding and convincing themselves of their own argument through the process of sharing their justification with others. At times the misconceptions and faulty arguments shared by a student were pivitol in the community gaining a deeper understanding of the mathematics.

Ball, Lewis, and Thames (2008) analyzed a six minute segment of Ball teaching a grade 3 class of students when one student proposed that they were thinking about the number six and whether or not it was even or odd, which started an engaging discussion among the students. Next, one child challenged the students to justify their thinking, and then the students began to make a progression of claims that included making claims, clarifying claims, and then other students proposing rewording and revising those claims. In just six minutes there were 30 claims made by students and 4 by the teacher. Ball, Lewis and Thames found that both making mathematical claims and making claims about mathematics were key mathematical activities in this classroom. There was evidence that students were aware of and practiced the following key conditions of successful classroom argumentation: “what counts as a mathematical claim, how to express mathematical claims so they are usable by others, and how to evaluate and respond to mathematical claims” (p.26).

Through this case study, Ball, Lewis and Thames (2008) also found evidence of the following key teacher moves that facilitated the classroom argumentation: “(a)provides mathematically engaging experiences for students to make claims about; (b) invites their claims about those experiences; (c) prompts them for sufficient elaboration of claims so that all can “dig into” them; (d) ensures that they hear one another’s discussion of claims; and (e) solicits other students’ reactions to the original claim”(p.32). The researchers also found that naming and defining mathematical terms was important in the formulation of claims. The students used the mathematical terms of “even” and “odd” and related properties as they made conjectures, asked for justification, and explained why they agreed or disagreed. Using names and definitions forms a common language that supports the class argumentation process.

Ryan and Williams (2007) research in the United Kingdom demonstrated the power of argumentation to enable students to talk about mathematics in a connected way to deepen conceptual understanding and to challenge and resolve their misconceptions. Based on this research they developed a helpful guide to common student misconceptions and a PD model to assist teachers in better understanding student misconceptions and how to challenge those misconceptions by involving students in argumentation. This research is described in more detail in the following section on teacher PD.

Teacher Professional Development

Much has been written about children’s understanding of integers and integer operations and teaching strategies for improving student understanding of integers; however, little is written about effective PD for teaching teachers to teach children about integers in a way that improves student understanding of integers and integer operations. Upon review of literature related to this topic, there appear to be recommended topics for the focus of teacher PD for improving student understanding of integers: Student misconceptions and student argumentation. What follows is a brief overview of the literature that informed the creation of the PD for *Jumpstart* teachers on the topic of integer operations.

Helping Teachers Understand Common Student Misconceptions

Including a Developmental Look at Misconceptions

A great resource for PD to help teachers better understand common misconceptions and errors students make at different ages is research conducted by Ryan and Williams (2007) who completed a large, cross-sectional survey of 15,000 children ages 4 to 15, in year 2005, in the U.K. They found that children learn and improve in mathematics slowly and plateau around the age of 11. For some reason, there is very little progress made between the ages of 11 and 14 years. The extra exposure to math curriculum, teaching, and assessment in secondary school seems to have little effect on their mathematics achievement. Ryan and Williams used data from the Mathematics Assessment for Learning and Teaching (MaLT) database, a standardized test given to a national sample of students (Williams, Wo, & Lewis, 2005). According to Ryan and Williams, as learners get older, their needs change, so teachers need to adjust instruction in order to meet the needs of their intellectually developing students.

What follows is a summary of the findings of Ryan and Williams (2007) specific to students' developmental learning of integers that show some of the common misconceptions at different age levels that represent errors related to conceptual problems and the percent of students who accurately solved these kinds of problems: 23% of 8 year olds recognize that subtraction is not commutative (p. 188); 44% of 10 year olds recognize and order negative integers (p.197); 45% off 11 year olds recognize negative decimal numbers on a number line (p.202); 25% of 14 year olds subtract integers and 44% divide integers correctly (p.218).

It is impossible to prepare teachers for every possible kind of student misconception; however, including sample misconceptions in PD can provide teachers with opportunities to brainstorm how they would adjust their instruction to address misconceptions. The PD can also provide practice creating questions that teachers can ask students to assess the cause of the misunderstanding and to advance their thinking without telling the students they are wrong.

Learning from Pre-Service Teacher Preparation Programs

Baroody and Coslick (1998), in their development of an Elementary Mathematics Methods textbook, created an entire section to provide PD to new teachers on the common misconceptions children have related to understanding integers. They emphasize the strange new world students find themselves in when they are introduced to signed numbers. Suddenly when they see $^{-}2$, it no longer means "take away two." Complicating this confusion of negative numbers with subtraction is the language many people use day-to-day to refer to negative numbers as "minus," such as when someone says that the temperature went down by "minus 2" degrees. To avoid reinforcing misconceptions and to emphasize their true meaning, teachers should refer to integers as positive or negative, not plus or minus (Baroody & Coslick, 1998).

Baroody and Coslick (1998) provide pre-service teachers with a short vignette that describes a young girl's only experience with negative numbers in the context of temperature. The student is then introduced to a *Pac-Man* (MiltonBradley, 1982) card game that includes positive and negative numbers. Six months later, when asked where the student might find negative numbers, the student responds in "Pac-Man" (Baroody &

Coslick p. 8.22). Below are reflection questions provided to the teacher following the reading of the vignette:

1. What does Alexi's experience with the *Pac-Man*TM game suggest about introducing children to negative numbers?

2. Other than temperature, accounting (credit and debit), and keeping score in games, are there other everyday uses of integers?

3. Mathematically what do integers tell us that whole numbers do not?

4. A student teacher reads the expression -6 + (+7) as "minus six and plus seven" and 6 – (+7) as "six minus plus seven." Is this good educational practice? Why or why not?

5. What is the answer to 5 – (-8) = ? How would you explain to a student the answer you got? What is the answer to -5 x (-8) = ? Why does it have the sign it does? (Baroody & Coslick, p. 8.22)

Students will ask teachers some of these same questions, and teachers need to be prepared to answer these questions with more than, "That is just how it is" (Baroody & Coslick, p. 8.22).

This past section provided some resources for PD for pre-service and in-service teachers to improve their ability to teach students about integers by focusing on identifying student misconceptions and instructional strategies to target these misconceptions. These resources were used in this study to inform the development of the curriculum, student assessments, teacher assessments, and PD activities for the Jumpstart program; however, what is missing is what students need to do and think to improve their understanding of integers. What follows is a review of literature on student discourse in mathematics classrooms and how to help teachers facilitate mathematical conversations in their classrooms.

Teacher Facilitation of Mathematical Conversations Using Argumentation

Argumentation is a classroom discussion method previously researched in the context of science and mathematics instruction (Enyedy, 2003; Kenyon, Kuhn, & Reiser, 2006; Kuhn, Kenyon, & Reiser, 2006; Ryan & Williams, 2007). Ryan and Williams (2007) support the use of argumentation in the math classroom based on the work of Bakhtin (1986). From this point of view, learning and teaching are thought of as communication, and mathematics learning as a certain kind of dialogue in which the rationale is mathematics argumentation. Ryan and Williams warn that not all good discussions will result in good mathematics. Common feature of discussions such as listening, responding, reflecting, and questioning occur in argumentation, but what must stimulate this conversation is the mathematics. The discussion could begin with a problem that needs to be solved, but argumentation is best supported when the problem induces different points of view or strategies to discuss and argue about. A rigorous mathematics discussion includes reasoning and persuasion using mathematics (Ryan & Williams, 2007).

A key finding of the research conducted by Enyedy (2003) that informs PD is that the models that students produced to support their claims served to make their reasoning visible. This allowed their reasoning to become an object of discussion and reflection. Teachers can facilitate classroom discourse by providing students with opportunities to make their understanding visible to promote discussion in small groups and in a whole class discussion.

Classroom Structures to Facilitate Argumentation

To better understand how to implement argumentation in mathematics, Ryan and Williams (2007) provide a process for preparing teachers to have classrooms of students who engage in argumentation about mathematics. In their research, 11-year old children were given a diagnostic assessment to determine areas where there were significant differences in responses. One of these items was selected as the problem to be the basis of an argumentation. Ryan and Williams worked with students in small focus groups. The emphasis of the discussion was on the student's reasoning and argumentation. As researchers in a teacher role, they facilitated the discussion by providing useful representations or models as needed to promote progress in the discussion. After their extensive work with focus groups, Ryan and Williams came up with several key ingredients for argumentation to occur.

First, according to Ryan and Williams (2007), it is important to begin with a problem that can be that can promote differences of opinion to discuss. Second, there must be class and small group norms that establish a community where children are able to safely voice different points of view and have others seriously consider their perspective. Third, the class must agree on and follow criteria for what makes a good argument (such as justifying one's reasoning with mathematics). Finally, there has to be time taken for the class or group to reflect on what has been discussed and learned and whether or not the discussion caused individual perspectives to change or to be confirmed. Based on Ryan and William's (2007) research, they determined that there was a dialogue cycle for argumentation that included four stages :

1. Articulation: “ I think that . . .”because . . .”
2. Re-formulation: “I listened to what X said and now I think that . . .”
3. Reflection: “My reasoning was/was not correct because . . .”
4. Resolution: “I now think that . . . because . . .” (p. 32)

Stigler and Hiebert (1999) found that students in their study often made the following argumentation moves : “alternatives, conflict, clarification, press on and so on” (p.35). Good mathematicians evaluate arguments others have made; they learn and discover new mathematics. So it is important for students to experience this same kind of metacognition and reasoning as they learn to be mathematicians through argumentation. There are norms that can be established in the classroom for what makes a “good” argument. Ryan and Williams (2007) provide the following sample list of expectations for discussion:

1. One person (with the pencil) speaks at a time, everyone else listens.
2. When you take the pencil from someone, begin your explanation or reason with: “I agree/disagree with . . . because . . .”
3. Then you can go on with: “So I think that . . .”
4. Make sure everyone has a turn to speak.
5. Leave time at the end of the session to help the reporter get ready. (p. 46)

Setting up these clear expectations allows all students to participate in an appropriate way to show respect for the opinions of others and also keep the discussion focused on the mathematics and reasoning. Ryan and Williams also recommend a group reporting practice to hold students accountable for listening. The following is a sample of a Mathematics Discussion Record template:

The question was ……………………………………
We discussed the following different answers:
We decided the correct answer was……
We decided this because …….
But did not agree with this because……
Signed……………………………… (reporter) Date: …(p. 46)

Students can take turns completing the discussion record and get assistance from the class in making sure that the discussion is represented appropriately.

Ryan and Williams (2007) have worked to prepare teachers to facilitate argumentation in their classrooms. They have used the term *general pedagogical strategies* to describe the following teacher moves that improve argumentation outcomes related to mathematics:

> Elicitation of a variety of alternative 'answers' and arguments, asking children to listen and sometimes to paraphrase others' views, seeking further clarification of arguments, helping to formulate and encourage a minority point of view, seeking support and dissent, criticizing the reasons and not the individual, and so on, as well as pressing for reasons or 'backing.'(Ryan :& Williams, 2007, p.43)

The work of Ryan and Williams (2007) and Enyedy (2003) has informed the development of the PD of *Jumpstart* 2010 for teachers. By understanding how to set up the class environment for argumentation and how to facilitate the discussion, the teachers set the expectation that students take responsibility for their learning and understanding.

Improving Teacher Content Knowledge Related to Integer Operations

In order to develop deep conceptual understanding of integers within students, teachers need to have a deep flexible content knowledge in the courses they teach. In work with pre-service and in-service teachers, several authors emphasize that teachers need to not only understand the facts and concepts of their discipline, but also the way the concepts are connected and the way members in their field go about creating new knowledge and determining the validity of new claims (Anderson, 1989; Ball, 1990; Borko, 2004; Borko & Putnam, 1996; McDiarmid, 1995; McDiarmid, Ball, & Anderson, 1989). To better understand this third option for PD, what follows is the research on content-focused PD and effective components of such a program.

Turnuklu and Yesildere (2007) gave pre-service teachers the following sample dialogue between a student and their teacher:

> Teacher: What is the result of -3 + 5?
> Student: -3 + 5 is - 8.
> Teacher: How did you do it?
> Student: 3 plus 5 is 8. The sum has the sign of the first integer. (Turnuklu & Yesildere, 2007, p. 9)

Turnuklu and Yesildere (2007) found that 44% of the teacher candidates understood the students' misconception in the dialogue, but 56% of them had their own misconceptions about addition and subtraction of integers. They did not understand the connection between addition and subtraction. Examples of their misconceptions follow:

> 5-3 is an addition operation. It means "5 plus -3.
> There is no difference between 5-3 and 3-5
> The set of integers is commutative under subtraction (Turnuklu & Yesildere, 2007, p. 9)

The teachers in the study tended to try to re-explain the task or ask questions to explain the task rather than trying to understand the students' thinking of their misconception. This response did not satisfy the students because it did not address their misconception. According to the work of Turnuklu and Yesildere (2007), teachers need to understand the difference between a sign and an operation and the importance of assisting students in understanding the difference.

According to a review of math and science PD programs by Kennedy (1998), "programs whose content focused mainly on teachers' behaviors demonstrated smaller influences on student learning than did programs whose content focused on teachers' knowledge of the subject, on the curriculum, or on how students learn the subject" (p. 18). In a research study of PD, a national sample of teachers was surveyed by Garet, Porter, Desimone, Birman, and Yoon (2001) who discovered that activities that are content focused but do not increase the knowledge and skills of teachers result in a negative impact on teacher practices. Therefore, just providing teachers with content is not sufficient; the PD must provide a deepening of the teachers' conceptual knowledge and skills in the subject(s) they teach.

Developing and Researching Professional Development

Effective Professional Development

A review of the literature on PD informed the design of the Jumpstart PD and the plan for studying the impact of the PD on teacher and student understanding. Research has shown that there are several key ingredients to effective PD. In developing the PD materials, it is important to keep in mind the need for flexibility that the facilitator and users may need (Borko, 2004; LeFevre, 2004; Remillard & Geist, 2002). Based on the context for implementation, there may be a need to adapt the PD to the needs of the teacher and allow the teachers to further adapt it to meet their needs. So researchers need to decide on the tradeoffs of studying fidelity of implementation and the elements of the PD that must be preserved to maintain the goals of the study (Borko, 2004). According to a review of research on mathematics teaching by Hiebert (1999), there are several components of effective opportunities for teachers to learn new teaching methods:

> (a) ongoing (measured in years) collaboration of teachers for purposes of planning with (b) the explicit goal of improving students' achievement of clear learning goals, (c) anchored by attention to students' thinking, the curriculum and pedagogy, with (d) access to alternative ideas and methods and opportunities to observe these in action and to reflect on the reasons for their effectiveness. (Hiebert, 1999, p. 15)

There should be an attempt to include a component of prolonged engagement of the participants, which can be costly in terms of time and resources. However, it may be as important as the chosen area of focus to the growth of teacher knowledge and improvement in the quality of instruction and student achievement.

Professional Development Design Considerations

Loucks-Horsley, Stiles, Mundry, Love, and Hewson (2010) have been conducting research on PD and reviewing research on effective PD programs for over ten years. In their latest edition (2010) *Designing Professional Development for Teachers of Science and Mathematics*, they share five important values for designing PD. First, students and student learning outcomes need to be the focus of PD, with an emphasis that these outcomes need to be for all students. Second, the design of the PD must address the PCK of science and mathematics teachers, not just focusing on pedagogy or content separately. The way the PD is enacted needs to be in line with the pedagogy expected of teachers for working with students. So if the PD is about inquiry instruction for students, then the design must inlude inquiry opportunities for teachers. Next, teacher leaders should be involved in the design and facilitiation of PD building capacity within the school and district in which they work. Finally, PD design must be aligned with and support systemic changes by addressing standards, assessment, and curriculum to move towards improvement in teaching and learning. What follows is some additional research related to four components of PD design: ensuring equity for all students, transforming teacher beliefs and practices, understanding possible outcomes, and ensuring relevance for teachers.

Ensuring equity for all students. Horizon Research conducted the 1993 National Survey of Science and Mathematics Education with a probability sample of 1,250 schools in the United States which included 6,000 teachers in grades 1-12 (Weiss, 1997). Horizon Research compared low and high ability high school students and the opportunity provided to them to participate in different classroom activities. The results showed that there were fewer opportunities afforded to low-ability students to engage in inquiry based science activities. These same students were also provided with fewer opportunities to write or talk about their reasoning while solving problems in mathematics classes. Therefore, PD should emphasize ways to close opportunity to learn gaps so that all students have the opportunity to engage in inquiry and high level thinking in mathematics (Weiss, Matti, Smith, 1994).

Transforming teacher beliefs and practices. For PD to be transformative, by changing deep seated beliefs of teachers, Thompson and Zeuli (1999) provide the following recommendations based on their years of work with teachers:

1. Engage teachers in the subject matter and how kids learn in ways that create a high level of cognitive dissonance and cause them to start thinking about better ways to teach.
2. Allow enough time and opportunities for teachers to think through this dissonance and engage in discussions with other teachers about it.

3. Use actual student work, video tapes, case studies (transcripts), or opportunities to engage as learners for teachers to experience the kind of instruction desired and to work through the related cognitive dissonance.
4. Provide opportunities for teachers to think and plan for how this new understanding will impact what they do in their classrooms and how their work with students.
5. Engage teachers in a cycle of ongoing reflection on practice and improvement. (Thompson and Zeuli,1999 p.356-357)

In order to transform teachers old ways of teaching, research by Little (1993) found that the PD experiences needed to allow teachers opportunities to learn through constructivist learning activities to understand deeply the importance of these kinds of experiences for their students.

Understanding possible outcomes. According to the extensive research of Ball and Cohen (1999) on teacher PD, professional developers need to have realistic expectation of outcomes related to teacher change. Their research has found that change in attitudes, beliefs, and instruction often come after teachers have used the new practice in their classrooms and have experienced positive results with their students. Therefore, the practice focused on in PD should be something teachers can implement immediately in their classrom.

Ensuring Relevance for Teachers. Mundry and Loucks-Horsley (1999) conducted a series of case studies at different grade levels as part of the National Institute for Science Education (NISE) PD project. In the case study of Riverside Middle School

(Urban Diverse School serving students in grades 6-8) they found that implementation of a new reform mathematics curriculum required that the PD was designed to encorporate a balance between pragmatically what teachers are concerned about such as being prepared to implement the new curriculum and the new pedagogical practice that was the focus of the reform effort. Therefore, the Jumpstart PD was developed around student activites to ensure such a balance for the teachers between ensuring they are prepared to teach as well as developing their PCK.

Data Collection to Measure Effect of Professional Development

Researchers recommend a theory of teacher change and a theory of instruction to determine whether the PD improves teachers' content knowledge and their quality of teaching (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Sloan, 1993). Data on teacher content knowledge prior to the PD and possibly several other times if the PD is ongoing should be collected. Additional implementation data such as classroom observations can also determine if there are changes in quality of instruction provided. It is important to provide a rich description of the PD experience. If it is not possible to conduct multiple observations throughout the year, it is important to collect additional data through structured and unstructured interviews with teachers or focus groups of teachers. Krueger (1988) and Krueger and Casey (2000) provide practical suggestions for structuring focus-group questions that can be used to develop the focus group interview. One suggestion to consider is to hold a brainstorming session with colleagues and/or teachers to generate questions (Mertens, 2005).

Many studies gather data on student performance at the beginning of the year and then after teachers experience PD and have time to implement what they have learned in the classroom. However, the results of an observational study by Harris and Saas (2007) showed that their content-focused PD for middle school mathematics teachers had an impact on the student performance the year following the PD program and did not have an impact the year the teachers were participating in the PD. Achievement tests may not measure the kind of information that relates to the PD. So additional options may need to be considered such as interviews with a sample of the students in each teacher's class, interviews conducted with a focus group of students from a teacher's class and work samples from the students.

Conclusion

According to Ryan and Williams (2007), it is not a matter of subject knowledge or subject knowledge only, it is a matter of knowledge about the best tasks, representations, models, contexts, and so on, to introduce in a given situation what Shulman (1986) called "pedagogy content knowledge" (p. 39). Based on this review of literature, the PD for this study integrated content knowledge development in teachers with a strengthening of their classroom pedagogy related to facilitating mathematical discussion in the classroom using argumentation.

The National Council of Teachers of Mathematics (1989) emphasizes the importance of students understanding integers and makes the following recommendation:

> Instruction should help students see the underlying structure of mathematics, which bonds its many individual facets into useful, interesting, and logical whole. Instruction should employ informal explorations and emphasize the reasons why

> various kinds of numbers occur, and relationships between number systems. (NCTM, 1989, p. 91)

The study of number theory and integers provides students with a rich opportunity to develop reasoning skills as they look for patterns and justify their thinking. Reasoning is emphasized in *Focus in High School Mathematics: Reasoning and Sense Making*. Reasoning and sense making are described as

> Student's ability to think about and use mathematics in meaningful ways. . . . Mathematical reasoning involves drawing conclusions on the basis of assumptions and definitions. Sense making involves developing an understanding of a situation, context, or concept by connecting it with other knowledge. (NTCM, 2009, p. 1)

This research study combined teacher reasoning about integers through argumentation during the PD to improve teacher PCK in order to impact student understanding of integers and integer operations. This study will add to the current research base in that it includes a larger sample size than most of the previous studies, and it includes a quantitative analyses along with qualitative analyses to better understand patterns in teacher understanding, whether there was a change in teacher PCK following the PD, whether there was a change in student understanding following implementation of the revised curriculum activities for the unit on integer operations, and whether this change in student understanding was statistically significantly different than the previous year.

CHAPTER THREE: THE PILOT STUDY

A Pilot Study to Explore Student Understanding of Integers

In May 2008, the researcher conducted a pilot study of student integer understanding. The goal of this study was to document an "average" developmental trajectory for negative integers, and to understand the onset and offset of counting, rule application, symmetry detection, categorical perception, and other processes. The expectation was that there would be large individual differences at younger ages with a steady convergence towards adulthood. Each student interview was audio recorded and transcribed. During the analysis of the interview transcripts, attention was paid to the results of any individuals who exhibited patterns that were well outside the "average" range. Based on teacher and student interview data, a comparison was made of children exposed to different curricula and instructional strategies. In combination, these two lines of work provided a foundation for subsequent intervention and development studies.

Research Question

How do children take a well-established knowledge structure for positive numbers and build on it in a way that goes beyond that structure to understand negative numbers?

Research Sub-questions

1. What is an "average" developmental trajectory for negative integers?
2. At what stages of "average" development is there an onset and offset of counting, rule application, symmetry detection, categorical perception, and other related processes?

3. How do the individual differences at younger ages compare to individual differences as a child nears adulthood?
4. How do children conceptualize negative numbers?
5. In what ways do children communicate their understanding of integers?

Methods

Upon consent of the principals of a middle and high school in Central Texas, mathematics teachers of students in grades 7, 9, and 11 were approached for permission to ask their students if they would be interested in participating in a study. A consent letter was sent home to the parents of interested students explaining the voluntary nature of their participation. Interested students were asked to sign a consent form. The goal was to include 24 students in the research project from each of grades 7, 9, and 11. The students were given permission to leave class for approximately 25 minutes to participate in a computer task comparing two integers to determine which integer was smaller or larger, followed by a short interview.

Participants

The prior school year, both the middle and high school had been rated as "Academically Unacceptable" according to the Texas Education Agency. Participants were recruited from mathematics classes. They returned consent forms signed by parents or legal guardians. The sample included 21 grade 7 students, 24 grade 9 students, and 23 grade 11 students, but only 20 of grade 11 students were able to complete the full interview. Table 1 provides the average State Mathematics Assessment (TAKS) math

Table 1   State Mathematics Assessment (TAKS) Performance for the Sample

	TAKS 2007 Average	TAKS 2008 Average
Grade 7	2159	2158
Grade 9	2073	2062
Grade 11	1980	2035

Note. TAKS scores are standard scores with 2100 representing passing interviews.

Students were provided with a 15-minute computer task to compare two integers to decide which was larger or smaller depending on the prompt. Following the brief integer comparison task on the computer, the students were interviewed to determine what strategies or mental representations they used to complete the task and to complete five integer arithmetic problems ($^{-}5 + 8$, $^{-}3 + {}^{-}6$, $2 - 7$, $^{-}2 - 5$, and $^{-}4 \times 5$) explaining their thinking or reasoning for each problem. The following questions were given to the students:

1. When you are comparing the numbers on the screen, how did you decide which one was larger[5]?
2. If you were not sure, how would you figure it out?
3. Here are a few computation problems. As you solve each problem, explain what you are thinking about or how you are solving each problem.
4. Is there something you could draw to help you solve one of these problems?
5. Where have you seen negative numbers before? In school? Outside of school?

[5] Students are typically taught that larger numbers are the ones further to the right of zero. They are exposed to inequalities which are referred to as "greater than" or "less than" signs. So larger in this situation is the same as greater.

6. Why do you think it is important to understand negative numbers?

Results

The transcripts were read the first time to develop a set of codes to use to identify the strategies and representations used by students to compare integers and to calculate the integer arithmetic solutions. An analysis of student responses to each question was conducted assigning each response a code. Another experienced mathematics teacher similarly read and coded each response. The codes were compared for discrepancies. Discrepancies were discussed and after a review of the transcript and student work, a final code was assigned. Some of the students gave responses that were unclear or filled with comments about confusion or lack of certainty. These were given an “other” code, because they did not meet the description of any of the other codes chosen. What follows is a table of data for each question and type of arithmetic problem along with a summary statement of observations from the analysis. Whenever appropriate, relevant information from the interactions with the teachers related to their instruction of integers was noted. Of the 23 students, only 20 were able to stay for the interview because three students had to go to the next class.

Student Question 1

What were you thinking about when you were comparing the numbers on the screen? How could you tell which one was larger? What if there were two negative numbers?

Table 2   Percent of Students Using Strategies for Comparing Integers

Strategy/Representation	Grade 7 (n = 21)	Grade 9 (n = 24)	Grade 11 (n = 20)	Total (n = 65)
Number line: Close to 0	29	42	45	38
Opposite of Positive numbers	10	17	25	17
Fact/Memory	4.5	4	5	5
Money metaphor	4.5	0	5	3
Misconception: Larger negatives (magnitude) are larger than smaller negatives (smaller magnitude)	14	0	5	6
Other	38	37	15	31

Note. Source: Analysis of student interview responses.

A majority of the students used a mental number line representation or the closeness of the number to zero to determine which of the two numbers on the computer screen was larger or smaller. A larger percentage of grade 7 (38%) and grade 9 (37%) students were confused, unsure, or unable to explain the strategy or mental representation they were using to compare the integers. The second most common strategy for grade 9 and grade 11 students was to use a rule such as thinking the opposite of what one would for positive numbers, which is that the smaller negatives (smaller in magnitude or absolute value) are actually greater than larger negatives (larger in magnitude or absolute value).

Student Question 2

If you were not sure, was there anything else that you did to decide?

Table 3   Percent of Students Using Alternative Strategies for Comparing Numbers

Strategy/Representation	Grade 7 (*n* = 21)	Grade 9 (*n* = 24)	Grade 11 (*n* = 20)	Total (*n* = 65)
#line/close to 0	29	37	30	32
Rule: Opposite of Positive numbers	5	13	5	8
Fact/Memory	9	0	5	5
Money metaphor	0	0	5	1.5
Misconception: Larger negatives are Larger	0	0	5	1.5
Guess	9	4	5	6
Other	48	46	45	46

Note. Source: Analysis of student interview responses.

The largest percentage of students did not have an alternative strategy or representation to use if they were not sure about which one was larger or smaller, as shown in Table 3. This is the group marked *Other*, because the students really were not sure about what to think about if they were not sure. The number line or closeness to zero was the second most common strategy for each grade level.

Student Question 3

Here are a few computation problems. What are you thinking about or how are you solving each computation problem?

Table 4 Percent of Students Using Strategies for Integer Addition Problem: $^{-}5 + 8 = ?$

Strategy	Grade 7 (*n* = 21)	Grade 9 (*n* = 24)	Grade 11 (*n* = 20)	Total (*n* = 65)
Movement on a number line	24	21	20	21
Rule: Keep the sign of the largest #	5	8	10	8
Reverse order and subtract	5	21	5	11
Cancellation with objects	24	17	10	17
Money metaphor	0	4	0	1
Finger count	14	16	0	8
Misuse of multiplication rule	14	8	5	9
Error: Keep the negative sign	5	0	20	8
Other	9	13	25	17

Note. Source: Analysis of student interview responses.

When adding $^{-}5 + 8$, an equal number of grade 7 students used the number line movement as action with objects (i.e., colored chips), as shown in Table 4. Three of the students drew a circle divided in thirds with a plus in one section and two negatives in the other sections. They referred to this as the “pie-man” or “China-man.” This is a memory strategy taught to them by their teachers for determining the sign when multiplying or dividing integers. Unfortunately, they were using it in the wrong context by applying it to addition of integers; one student after drawing the “pie-man” wrote an M and D at the top and realized that it should not be used for this problem. She then corrected her error. The strategies used most by the grade 9 students (21% each) were to use the number line or to reverse the order of the integers and subtract (8 -5). Overall, students using the number line were the most accurate on this problem.

Table 5   Percent of Students Using Strategies for Integer Addition Problem: $^{-}3 + {}^{-}6 = ?$

Strategy	Grade 7 (*n* = 21)	Grade 9 (*n* = 24)	Grade 11 (*n* = 20)	Total (*n* = 65)
Rule: Add and keep negative	9	29	10	17
Objects (i.e., take more away)	9	25	5	14
Accurate number line movement	29	4	5	12
Inaccurate number line movement	9	4	10	8
Misuse of multiplication rule	39	13	50	32
Other	5	25	20	17

Note. Source: Analysis of student interview responses.

The majority (32%) of students misapplied the rule for multiplication on this problem responding with the answer of positive 9. This was seen when grade 7 students drew the “pie-man” and grade 11 students said things like “a negative and a negative makes a positive.” Less of the grade 9 students fell into this trap. A greater percent (25%) of grade 9 students used objects for this problem. Students in Algebra 1 were involved in daily warm-ups using integer tiles (red/black) for doing integer operation problems and for solving one and two step algebra equations with integers.

Table 6 Percent of Students Using Strategies for Integer Subtraction Problem: 2 – 7 = ?

Strategy	Grade 7 (n = 21)	Grade 9 (n = 24)	Grade 11 (n = 20)	Total (n = 65)
Subtract, keep -	5	0	5	3
Slash & Dash- add negative 7	5	0	10	5
Fact memorized-no strategy	10	13	0	8
Story	0	4	5	3
Objects	10	4	15	9
Accurate use of # line	14	25	10	17
Inaccurate use of # line	14	0	0	5
Error: Commutative	23	29	40	31
Misuse of multiplication rule	14	0	0	5
Other (confused, unsure, unclear)	5	25	15	14

Note. Source: Analysis of student interview responses.

When given the problem 2 –7 = ?, the majority of students (31%) quickly responded with the number 5 treating the two numbers as positives and just subtracting, not realizing that the seven was larger which should result in a negative answer, as shown in Table 6. The majority of students who accurately solved this problem used a number line appropriately. A few of the students were taught a technique common in high school that is sometimes called “Slash and Dash” where the subtraction is changed to adding a negative (2 + $^{-}$7). This method as well as action with objects also resulted in accurate answers. grade 9 students had the largest percentage (25%) of students who were confused or unsure.

Table 7 Percent of Students Using Strategies for Integer Subtraction Problem: -3 – 5 = ?

Strategy	Grade 7 (n = 21)	Grade 9 (n = 24)	Grade 11 (n = 20)	Total (n = 65)
Rule: Add and keep negative	5	8	0	5
Slash & Dash-convert to addition	5	8	10	7
Fact-memorized	0	8	5	5
Objects referenced (i.e., chips)	24	4.5	0	9
Accurate number line movement	14	13	20	15
Inaccurate number line movement	9.5	0	5	5
Error: Subtract, and keep negative	14	4.5	45	20
Error: Use of rule for multiplication	19	29	0	17
Other	9.5	25	15	17

Note. Source: Analysis of student interview responses.

When asked to solve -3 – 5 = ?, the largest percentage of grade 11 students subtracted the numbers and kept the answer negative unaware of why except that they should subtract, as shown in Table 7. A large percentage of grade 7 and grade 9 students misapplied the rule for multiplication. More grade 7 students used objects (chips) with this problem modeling a strategy they were taught by making a key (showing which chips represent positive and which represent negative); however, some of them resulted in errors because they were starting with 3 negative and wanting to take away 5, so they needed 2 more negatives to take away.

Table 8 Percent of Students Using Strategies for Integer Multiplication: $^{-}4 \times 5 = ?$

Strategy	Grade 7 (*n* = 21)	Grade 9 (*n* = 24)	Grade 11 (*n* = 20)	Total (*n* = 65)
Accurate Rule Use	38	50	60	49
Keep Negative not sure why	14	4	0	6
Use of objects	14	0	0	5
Inaccurate Rule Use	10	13	0	8
Inaccurate use of number line	5	0	0	1.5
Error: Multiply then subtract number	5	0	0	1.5
Multiply as if positives 4 x5	5	16.5	5	9
Other (unsure, confused, unclear)	9	16.5	35	20

Note. Source: Analysis of student interview responses.

The majority of the students accurately used the rule which was typically stated as "a negative and a positive makes a negative" or shown with the "pie-man" memory device, as shown in Table 8. The most common mistake was to multiply the numbers as if they were positive resulting in an answer of 20. There was a large percentage (20%) of students who were not sure if the answer was positive or negative and they did not have a strategy or representation that they could use to verify which one it should be.

In terms of accuracy, grade 9 students outperformed the students of grades 7 and 11 (see Table 9). This may related to a warm up activity solving integer expressions and equations that these grade 9 students completed daily for a month prior to this interview that involved use of a manipulative for positive and negative numbers called Algebra Pieces (Math Learning Center , 2009)[6]. This activity was developed by the Algebra 1 team of teachers at the high school that the grade 9 students attended. The grade 11 students had a lower than expected accuracy; however, this student group had a much

[6] Burton (2006) provides more information about use of this manipulative.

lower average TAKS math score than the students from the other two grades. So these students may have had inadequate foundations in math.

Table 9 Overall Percent Accuracy of Integer Arithmetic

Accuracy	Grade 7 (n = 21)	Grade 9 (n = 24)	Grade 11 (n = 20)	Total (n = 65)
-5 + 8 = 3	62	100	60	74
-3 + -6 = -9	57	79	45	60
2 – 7 = -5	19	63	45	42
-3 – 5 = -8	19	25	35	26
-4 x 5 = -20	76	75	75	75
Total Accuracy	47	68	55	57

Note. Source: Analysis of student interview responses.

The last three questions were asked to determine what representations students were able to use to confirm their answer: their awareness of real world applications of integers, and their opinion on the usefulness and importance of having an understanding of integers and integer operations. These questions were asked to determine what additional understanding of integers students had beyond their knowledge of comparing and ordering integers and integer operations, especially any relevant out of school knowledge that they possessed.

Student Question 4

Is there something you could draw to help you solve one of these problems?

The students drew or created representations that were aligned with what the students had been taught which were number lines and chips (see Table 10). More of the grade 11 students were unsure or unable to draw anything than students in grades 7 and 9. One question remains, whether the students in grade 11 would have used chips or counters if they were provided to them.

Table 10   Percent of Students Using Representations to Solve Problems

Representation	Grade 7 (n = 21)	Grade 9 (n = 24)	Grade 11 (n = 20)	Total (n = 65)
Metaphor/Story: $	0	4	10	5
Sticks	0	8	5	5
Circles/Chips	38	42	25	35
Number line	43	46	35	42
Pie-Man	5	0	0	1
Other	5	0	5	3
Unsure	9	0	20	9

Note. Source: Analysis of student interview responses.

Student Question 5

Where have you seen negative numbers before? In school? Outside of school?

The juniors had the widest variety of places that they had seen or heard of negative numbers. Students from grade 7 had very limited exposure to contexts that use negative numbers. One question to consider for future research: Would exposure to real world application of integers in a variety of contexts improve integer understanding and accuracy in integer arithmetic?

Table 11   Number of Students with Different Locations of Integer Applications

Location	Grade 7 (*n* = 21)	Grade 9 (*n* = 24)	Grade 11 (*n* = 20)	Total (*n* = 65)
Math class	15	12	11	38
Bank, owe money	2	2	1	5
Making change	1	2	1	4
Stocks	0	0	1	1
Maps	0	0	1	1
Science	0	2	3	5
Temperature	3	1	1	5
Sports (yardage in football)	0	1	1	2
TV	1	1	3	5
Computer Graphic Design	0	1	1	2
Car Battery	0	1	0	1
I don't know	0	3	1	4
Paycheck deductions	0	0	1	1
Forensic Blood Count	0	0	1	1
Air Conditioner?	0	0	0	0
Total	23	26	27	76

Note. Source: Analysis of student interview responses.
No percentage is given due to multiple responses of students.

Student Question 6

Why do you think it is important to understand negative numbers?

For most students, negative numbers are associated with school and money (see Table 11). The students seem to agree that understanding negative numbers will help one in school, in real life when shopping to make sure the correct change is given, and in the future on the job. Several students felt more like it was only important because the teacher told them so or just to pass tests in school but that in reality negative numbers were not necessary anywhere else. Additional exposure to real world contexts for integers might allow students to better understand their importance.

Table 12   Percent of Students Sharing Reasons for Importance of Negative Numbers

Reason	Grade 7 (*n* = 21)	Grade 9 (*n* = 24)	Grade 11 (*n* = 20)	Total (*n* = 65)
Teacher told me	0	0	5	2
Banks, owe $, bills	14	12.5	15	14
Store, not get short changed	29	12.5	15	18
To better understand them	0	8	15	8
Temperature	5	4	10	6
Football	0	0	5	2
Life, future, job	9	12.5	25	15
Do math better	24	17	5	15
Other	5	12.5	5	8
I don't know	9	21	0	11
It's not important	5	0	0	2

Note. Source: Analysis of student interview responses.

Discussion

The primary goal of the pilot study was to evaluate the development of the mental representation of integers through middle and high school. The results suggest that grade 7, 9, and 11 students behave according to the rule augmentation hypothesis, using a combination of rules and a number line representation. In this regard, they are more similar to grade 6 students (Varma & Schwartz, 2008) than adults previously studied (Varma et al., 2007). However, this sample of grade 11 students had relatively low TAKS test scores in math so they may not be representative of the development of most grade 11 students. Additional research with a larger sample is needed to determine if this development hypothesis is correct.

Re-Analysis of Pilot Study Data for Grades 7 and 9

Prior to the development of the curriculum unit for this study, a review of the statements made during the interviews of the 7th and 9th grade students was conducted to determine areas in need of attention. Since the *Jumpstart* students are in grade 8, this data should be most similar to the kinds of thinking that they may exhibit. It appeared that the main difference in the student thinking related to an understanding that was rule based or an understanding that was connected to either a representation or real world context.

When the responses for these items were analyzed, there were five types of strategies that were coded as "Rule Based" and four types of strategies coded as "Connected." Table 13 and Table 14 provide representative statements from the students in the pilot study of these kinds of strategies.

Table 13 Representative Statements for Rule-based Understanding

Strategy	Example
Rule	"(2 – 7) It is going to be negative because 2 is smaller than 7, so negative 5.
Commutative	"2 – 7, this is like 7-2 which is 5. I learned that in second grade."
Subtract	"When you are subtracting, you just subtract (Student writes -3 -5 = -2)."
Misuse of X and ÷ Rule	"(-3 – 5) I'm thinking it should be just a 2, because it's like another negative sign, but a negative and negative is a positive, so that would make it just a 2."
Slash & Dash	"2 subtract 7? Well, what we do is slash and dash like that. (Student changes 2 – 7 to 2 + -7) Negative five."
Fact	"2 – 7 = -5, I just know it. I don't know how to explain it."

Note. Source: Analysis of student interview responses.

Table 14   Representative Statements for Connected Understanding

Strategy	Example
Story	“2 – 7, if I have food I want to eat less, so if I had 2 things but I really wanted to eat 7, I'm going to be starving for like 5 apples.”
Objects	“(Students drew 2 white squares and 7 shaded squares) I shade them so I can tell the difference. (Students circled 2 pairs of white and shaded squares). Two minus seven is negative five."
Number Line	“I think I just think of this as a negative number, (Student draws a number line and moves to the number 2). Two. So count to 7, one , two, three, four, five, six, count to seven, and you get the distance which is -5 (moves on number line from 2 to -5).”
Misuse of Number Line	“2 – 7, (Student draws number line showing positives only) instead of going forward 2 to 9 you go back 2 to 5.”

Note. Source: Analysis of student interview responses.

The data from 20 students from each grade level (7th and 9th) for subtraction of integers were analyzed to determine the prevalence of rule-based and connected understanding. Tables 15 and 16 provide the data from their responses.

Table 15 Percent of Student Responses for Integer Subtraction Problem: “2 – 7 = ?”

	Rule Based				Connected Understanding					
Grade Level	Rule	Commute 7-2	Misuse Rule for x & ÷	Slash & Dash 2+ $^{+}$7	Fact	Story	use of objects	use of # line	Misuse of # line	Other
Grade 7	5	25	15	5	10	0	10	15	15	0
Grade 9	0	35	0	0	15	5	5	25	0	10

Note: Source: Analysis of Student Interview Responses (20 student from each grade level)

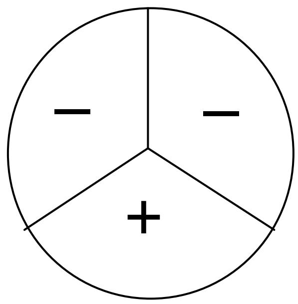
Table 16 Percent of Student Responses for Integer Subtraction Problem: “$^{-}$3 – 5 = ?”

	Rule Based				Connected Understanding					
Grade Level	Rule	Subtract #’s	Misuse Rule for x & ÷	Slash & Dash $^{-}$3+ $^{-}$7	Fact	Story	use of objects	use of # line	Misuse of # line	Other
Grade 7	5	15	20	5	0	0	25	15	10	5
Grade 9	10	5	35	10	10	0	5	15	0	10

Note: Source: Analysis of Student Interview Responses (20 student from each grade level)

Notice that only two strategies (commute and use of number line) were employed by more then 20% of the grade 7 and 9 students, respectively. This suggests that the students used a range of strategies to solve these sorts of problems—both before (as represented by the grade 7 students) and after (as represented by the grade 9 students) instruction. Of the two strategies that were used most frequently, students in both grade levels used one strategy—commutativity—relatively frequently. When using this strategy, students apply the commutative property—a property of multiplication and addition—to solve subtraction problems. In this case, that means they solved for 7-2 when asked to solve for 2-7. The researcher suggests that this misapplication of the commutative property implies that the students lack a conceptual understanding of integers and integer operations: Why does commutativity work for some operations but not for others?

Similar to the previous subtraction problem, -3 -5 resulted in a diversity of student responses. One strategy that emerged at both grade levels was the misuse of the rule for multiplication and division of integers. An example of the misuse of the rule for multiplication is shown in Figure 2. This image represents a memory strategy that many teachers introduce to students to help them determine whether a multiplication problem should result in a positive or negative answer. However, it does not apply to subtraction problems. Using this device, three students incorrectly determined that ${}^{-}3 - 5 = {}^{-}2$ by recognizing it as a subtraction problem, subtracting 3 from 5, and then determining that the sign of the answer is negative. Figure 2 provides a student response to further describe this strategy.



Student Response:

"I used this" (Student points to drawing)
"Negative and positive, I went like this"
(Student covered the negative and positive
signs with two fingers) "and then you have
negative."

Figure 2 Example of the misuse of a memory device for multiplication/division of integers

This is an example of students using a rule without any underlying connection: They learned this rule, but then they misapplied it because they did not have a deep conceptual understanding of why the rule is true. This finding suggests that PD should include discussions about the consequences of memory strategies and tricks that are taught without any conceptual connection.

Table 17 combines the analyses shown in Tables 15 and 16 in order to highlight the percent of students with each type of understanding. The following Table 18 reveals that the majority of students solved these problems using a rule-based understanding approach.

Table 17   Percent of Students with Each Type of Understanding

Problem: 2 – 7 = ?	Grade 7	Grade 9
Rule-Based Understanding	60	50
Connected Understanding	40	40
Other	0	10
Problem: -3 – 5 = ?		
Rule-Based Understanding	45	70
Connected Understanding	50	20
Other	5	10

Note. Source: Analysis of student interview responses.
Responses represented for 20 students per grade level.

Findings and Instructional Implications

Many students struggle with understanding operations with integers. The results of this study show that even after instruction with hands-on manipulatives and number lines, students continue to have difficulties with these subtraction problems and with explaining their understandings. Moreover, the majority of the strategies used relate to a rule-based understanding without a conceptual connection. This study reveals that not only do students struggle with the concept of subtraction; they do so in a variety of ways, that is, there was a large diversity of student strategies for solving these problems. This diversity suggests that an instructional approach is needed to challenge a variety of strategies and confront these misconceptions.

Argumentation was proposed as a useful instructional approach for exploring integers and integer operations, that is, engaging in argumentation around this content could enable students to discuss their diverse strategies—and the strengths and weaknesses around each of them.

CHAPTER FOUR: METHODOLOGY AND STUDY DESIGN

Study Background

The purpose of this project is to contribute to an urban school district by meeting the need of improving instruction and student achievement related to operations with positive and negative numbers using research-based instructional practices and curriculum for a summer three week program, called *Jumpstart*, for grade 8 students. The district Secondary Mathematics Curriculum Supervisor in collaboration with this researcher determined there was a need for this study based on the student performance results for the past two years of the program showing little improvement in the area of integer operations and a loss in performance of some of the students in classrooms taught by first year teachers. Table 18 provides student data from Jumpstart 2009 pre and post student assessments to demonstrate that positive and negative numbers was an area that was in need of improvement, which was the basis for determining the need for the work of this research project:

Table 18 Jumpstart 2009 Pretest and Posttest Results

Topics:	Pretest (*n = 206)*	Posttest (*n = 242)*	Growth *(n = 177)*
Positive and Negative Numbers	43%	49%	+6%
Patterns	40%	50%	+10%

Note. Growth is based on data for students who took the pre and post assessment.
Source: Jumpstart program data for 2009.

In general, the students and teachers were positive about the Jumpstart 2009 program and its curriculum and instruction components based on a survey given to teachers and students at the end of the program. However, student achievement was an area in need of improvement. In reflection, the PD was more focused on ensuring the teachers understood the activities, and less about how to facilitate the kind of classroom discourse that leads to improved understanding. Jumpstart is only a 15-day program; however, there are ways to improve instruction through PD for teachers.

For example, one of the activities for positive and negative numbers involved the use of a manipulative called "integer pieces," (Math Learning Center, 2009) which teachers were to use for two days to allow students to explore adding, subtracting, multiplying and dividing integers with these tools. At the end of the summer program when the teaching materials were returned, many of the integer piece tool packets were still in their original state—unopened. It was clear that they had not been punched out and used by the students. Perhaps this was due to the limited amount of time spent on PD for the teachers in the use of this tool. Another component of the positive and negative instruction was the use of a number line card sort activity, which explored addition and subtraction of integers. This was part of The America’s Choice Navigator: Positive and Negative Numbers (2009) curriculum. The method of subtracting integers on a number line used a vector model of directed distance between two points, as shown in Figure 3.

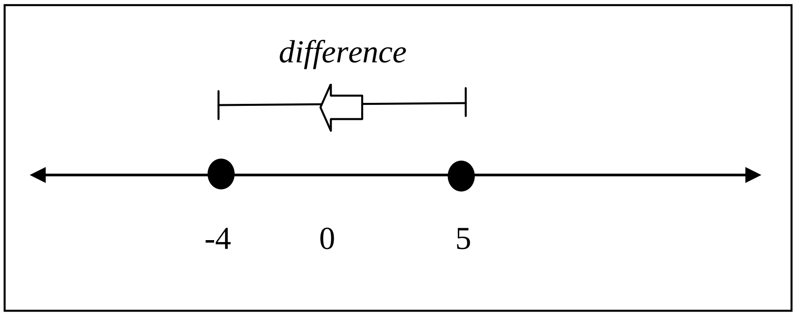


Figure 3   Number line Model for Subtraction as Directed Difference

Many of the teachers reported that this was a challenging representation to understand. Additional PD in this model for integers could improve teacher and student understanding of integers with this representation. Prior student performance data, teacher feedback, and a review of literature on research-based instructional strategies for integer operations led to the revision of the Integers Unit for the *Jumpstart* 2010 program and the development of an improved PD module for the teachers. The following research questions relate to measuring the extent to which these changes in the program resulted in statistically significant changes in teachers and students.

Research Questions

The following research questions will be addressed to determine the extent to which there is a relationship between teacher PCK and student understanding related to integer operations:

1. What are the general patterns of teacher content and pedagogical knowledge of integers based on responses to a pre and post assessment?

2. To what extent does PD impact teacher content and pedagogical knowledge as measured by growth between pre and post assessment?
3. Is there a statistically significant difference between the growth between students' pretest and posttest scores for Jumpstart 2010 compared to the growth made by students in Jumpstart 2009?
4. Do differences in teacher content and pedagogical knowledge explain more of the variance in student performance (pretest/posttest) than years of teaching experience of a teacher?
5. What is the relationship between teacher responses to the Constructivist Learning Environment Survey (CLES) and the student responses to the CLES?

Study Design

Participant Recruitment

Upon consent of the district Secondary Mathematics Supervisor, the research department, and the consent of the Jumpstart Principals, the teachers hired for Jumpstart 2010 were approached for permission to participate in the study by gathering the data from their teacher content and pedagogy assessments to be completed during PD and from their *Constructivist Learning Environment Survey* (Taylor, Fraser, & White, 1994). An effort was made to recruit a focus group of teachers to be asked more open-ended questions related to their experience with the Jumpstart 2010 integer lessons and PD. However, there was no interest in participating in a focus group; many of the teachers had commitments after their school day not making participation possible. Eventually, two teachers agreed to participate, but one became sick and was absent with a substitute when the interview was scheduled and the other had a family emergency to take care of on the day of the interview. Attempts to reschedule the interview did not work out in the teachers' schedule. Therefore, focus group teacher interviews were not included in this study.

Due to a potential conflict of interest since the researcher worked for district at the time of the study, all data collection and consent forms were distributed and collected by a local university graduate student. The graduate student provided the researcher with the data and consent forms at the end of the program when the researcher was no longer employed by the district. All data were de-identified by the grad student and the district supervisor prior to analysis by the researcher to protect the privacy and confidentiality of the teacher and student participants.

The graduate student gave a presentation in each classroom explaining the purpose of the study and the participating teachers sent home consent letters for their students to participate by allowing the data from their pretest and posttest to be gathered by the study researchers, providing information from a short survey of their experience with the integer unit, and for data to be gathered from any students volunteering to participate in a focus group. The teachers were asked to emphasize the voluntary nature of the students' participation and the protection of their privacy by using random student study identification numbers for their assessment and survey data. Interested students were asked to sign an assent form once their parent had provided consent for their participation and prior to completing the *Constructivist Learning Environment Survey*

(Taylor et al., 1994). The assessments were part of the Jumpstart 2010 program, so all students, regardless of consent, completed the assessments. However, the results were entered into a spreadsheet and de-identified by the district at the end of the *Jumpstart* 2010 program for analysis of the results in comparison to the student performance for *Jumpstart* 2009, which was also de-identified. Only the final score was provided for students. For students with consent, the actual integer assessments were collected and items analyzed individually by integer operation and for strategy use.

Participants

There were 341 students and 22 teachers in the *Jumpstart* 2010 program[7]. For this study, 21 of the 22 teachers attended the PD and consented to participate. One of the teachers did not attend PD and was replaced by a substitute half way through the program, so these two teachers did not participate in the study. Of the 341 students, only 102 had both parent consent and student assent complete. The data for these students were analyzed in more detail based on the information agreed on in the consent forms. Table 19 provides descriptive statistics for the sample of participating teachers based on data gathered with permission for this study.

[7] The school district provided de-identified student performance results for the pretest and posttest total score on the *America's Choice Navigator* Positive and Negative Numbers Assessment for the students enrolled in the program in 2010 and for the 315 students enrolled in the program in 2009 as a comparison group. Random identification numbers were assigned to the teachers for each year of the program and associated with the students in the data file for analysis.

Table 19   Characteristics of Jumpstart 2010 Participating Teachers (N = 21)

Variable	Minimum	Maximum	Mean	Standard Deviation
Age (years)	21	63	34	11
Gender: Female (%)	---	---	62	50
Teaching experience (years)	1	31	7.2	8.7
Prior experience teaching Jumpstart (%)	---	---	38	50
Advanced Degree (%)	---	---	24	44
Mathematics Major (%)	---	---	10	30

Note All characteristics are described as proportions unless otherwise stated (i.e., years).

Materials: Curriculum Revision

For the purpose of this study, the curriculum for the integers unit of the Jumpstart 2010 program was revised based on a review of research literature on improving student understanding of integer operations (provided in Chapter 2). During the development of the unit, feedback was provided on improvements that could be made by university professors, graduate students, teachers in the district (including those who taught in Jumpstart in the past), and the district Secondary Mathematics Department Supervisor. The goal was for this revised unit to develop the concept of operations on positive and negative numbers in a way that connects the concepts to a real world experience and a number line representation that is comprehensive in its application to all operations of integers. The number line revised unit for integers incorporated the use of vectors on a number line that integrated an application called NUMLIN on the TI-73 calculator and student activities written by Browning and John (1999) in a book entitled *Walking the Line: Activities for the TI-73 Number Line*, which are also publicly available on the Texas Instruments website. Figure 4 provides an example of how multiplication of integers is explored with the calculator in these activities, where multiplication is represented as repeated addition or subtraction.

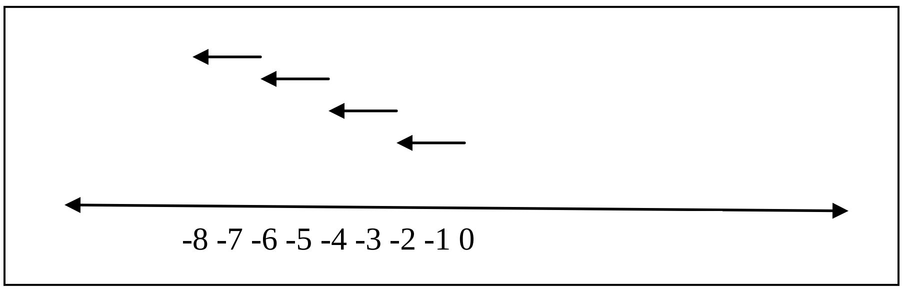


Figure 4 Number line Model for Multiplication of 4 x $^{-}2$

The students discovered the way operations on integers work by manipulating the vectors on the TI-73 calculator number line. The *Jumpstart* curriculum also included activities from the *Navigator: Positive and Negative Numbers* (America's Choice, 2008) curriculum module. Experienced teachers wrote the *Navigator* curriculum. The teachers' guide provides examples of common student misconceptions. There are also activities for students to look over the work of fictitious students to determine if they agree or disagree with their solution strategy. This forms the topic of class discussion to come to a consensus on whether the solution is correct to determine the effectiveness of the strategy. The teacher's guide provides examples of student misconceptions to prepare teachers for what they might encounter in the classroom. An example of how this curriculum provides PD for teachers in the area of misconceptions follows:

> Sometimes students learn ‘shortcut’ rules for addition (and subtraction) as isolated, mathematical procedures, which they over-generalize and, as a consequence, apply inappropriately. Students need to develop a sound understanding of addition (and subtraction) as directed distances on the number line before they can confidently use working rules. (America’s Choice, 2009, p. 35)

Another way the curriculum provides teachers with insights into student misconceptions is through Unit introductions that list the mathematical goals for the next few lessons and then the misconceptions that are addressed. An example of the misconceptions provided for teachers prior to students learning to subtract on a number line follows:

> Confusion about how to find a directed difference on the number line: Students think they can find the direction of the difference by moving along the number line from the first number in the expression to the second; for example, 8 – (-2) = -10.Using an incorrect “counting back” method by locating the first number on the number line and counting back in the wrong direction; for example, (-9) – 3 = -6.Commuting the subtraction, for example 3 – 5 = 5 – 3 = 2 (America’s Choice, 2009, p. 29)

The curriculum begins with a pretest followed by an introductory lesson for the students, where the students read several statements about integers made by a group of fictitious students. They have to decide which students they agree with and which they do not. This provides teachers with an opportunity to see which students agree with common misconceptions so that they can be addressed through this unit. When the teacher checks the students’ pretests, there is a scoring guide that teachers can use to note the misconceptions that students are having. The multiple choice items are pre-coded with misconceptions, and the open-ended questions have examples provided for teachers of common mistakes. Teachers can see the improvement their students make from the pretest to the posttest. The assessments provide teachers with information they can use to tailor the instructional activities and their attention to the needs of the students. These pre and post assessments were used for the Jumpstart program as well as for data collection for this study to measure student improvement in understanding of integers and integer operations. The integer unit overview is provided in Appendix A. Periodic journal activities were written for students to practice responding to misconceptions. Appendix B provides the sample misconceptions prompts that were used.

Teacher Professional Development

Based on the findings of the 1993 National Survey of Science and Mathematics Education (Weiss, 1997) that low-ability students were provided with fewer opportunities to engage in inquiry activities or to write about their reasoning while solving math problems, the PD and curriculum unit for this study was designed to ensure that the low-performing grade 8 students enrolled in Jumpstart 2010 would be afforded these kinds of opportunities and that their teachers would be prepared to facilitate such experiences.

The PD design was informed by the literature review conduted by Loucks-Horseley, et al. (2010) which found that effective PD programs focus on improving learning outcomes for all students, account for and advance teacher PCK, are implemented using the same pedagogy expected of teachers, involve teacher leaders within the school/district to buid capacity, and are aligned with the school/district standards, assessment, and curriculum.

Teachers participated in six hours of PD that focused on improving PCK around integer operations and the use of argumentation to facilitate student conversations about mathematics. A complete outline of the PD is provided in Appendix C. The morning started with registration and a light breakfast for the teachers while the graduate student described the purpose of the study and the data to be collected. At that time, consent forms were passed out and the pretest for teacher PCK was given to each consenting teacher.

There were 18 teachers present at the start of the PD session who consented and took the pretest. Three other teachers arrived an hour into the PD and were not given the assessment. However, all 21 teachers took the post assessment given the last week of the Jumpstart 2010 program. There was one additional teacher who was hired late and missed the PD. This teacher was replaced by a substitute after the first week of the program. This teacher was not included in the study. The teacher PCK assessment is provided in Appendix D. Once they had completed the assessment, the graduate student asked if they would be willing to complete a demographic survey. All consenting teachers completed a demographic survey, which is provided in Appendix E.

The design of this study was also informed by the case study of Riverside Middle School conducted by Mundry and Loucks-Horsley (1999) which documented that implementation of a reform mathematics curriculum required PD that was a balance between the practical issues that concern teachers such as being prepared to implement curriculum activities and the new instructional practices and beliefs that were part of the reform curriculum. Therefore, the student activities were used as the basis for developing the PCK of the teachers.

For the first part of the PD, the researcher asked a series of questions to access teachers' prior knowledge of teaching about integers, common student misconceptions, and any prior experience with students using argumentation about mathematics. A brief overview was provided to the teachers about the Jumpstart program and expectations, the performance data from Jumpstart 2009¸ and the changes made to improve Jumpstart 2010. Then the teachers all read a chapter from *Children's Mathematics 4-15: Learning from Errors and Misconceptions* by Ryan and Williams (2007) entitled "Children's Mathematical Discussions." This first section provided a transcript of a class discussion involving a problem where students were to compare two decimal numbers, one that was in the tenths and one that was in the hundredths. After sharing a few thoughts from the reading, the teachers were divided into two groups to read the next two sections. The first section transcript showed the teacher facilitating the discussion by asking questions, starting the discussion with a number line representation and asking a more resistant student to come up and put the decimal numbers on the number line which started a productive discussion with students making claims and justifying their thinking. The teacher allowed students time to think and did not move quickly to a resolution. She also pressed students to show their thinking. The transcript modeled a cycle of thinking that eventually led to students coming up with the same solution. During the discussion of the first reading selection, the teachers discussed these key findings from reading of the transcript.

The second reading selection started with a teacher sharing a common concern that students would learn misconceptions by being exposed to other students' misconceptions. During this part of the discussions, teachers shared different experiences and strategies for handling this concern. One theme emphasized by the reading is the kind of classroom environment needed for this kind of safe argumentation which is accepting of wrong answers, and where misconceptions are an opportunity to learn together. A collaborative classroom environment where it is common practice to share ones thinking and question the thinking of others should reduce the chance of students learning misconceptions from other students.

The design of the PD for this study was also based on the guidance provided by Thompson and Zeuli (1999) for creating a transformative learning experience for teachers. To create cognitive dissonance, several problem were provided for teachers to engage in reasoning and sharing of their thinking with others that would bring up questions and uncertainty that would need to be resolved through inquiry and discussion, and to provide sufficient time for that process to occur. The next part of the PD allowed teachers to work in groups on an integer operation problem. They could choose between representing it with a story, a rule, a number line model, a chip or tile model, or other representation. Then a discussion was held where groups had to connect their model with another group's model and argue as to what made it more effective or better in solving problems all the time. An engaging discussion broke out when two groups were arguing that the number line was fine for addition, but it was not a good model for subtraction.

The next two parts of the PD covered addition and subtraction of integers on a number line using the America's Choice *Navigator* (2009) number line models. The teachers understood how to do addition on a number line, but when asked to show subtraction, there was heated discussion in many of the groups. One group even sent a member to go to each of the groups to see what they thought was the best way to represent the given problem. The discussion that followed lasted almost an hour as teachers struggled with the meaning of representing subtraction on a number line, whether their way would work all the time or just sometimes, the challenges they felt students would have with the model. However, in the end they seemed to agree that it might really make sense to students. Before sharing out as a whole group, the expectations were set for the discussion modeling what should happen in their own classrooms for a safe place for argumentation. Since measuring what occurred during PD was not part of this research study, and teacher consent was not obtained to share their responses during the PD, no more information can be provided at this time. After lunch, the teachers explored the calculator activities with TI-73 NumLn application and continued to argue about the different representations for subtraction and multiplication of integers. Then in the final part of the PD where they explored some of the small group-center activities their students would be doing and discussed which parts or questions of these activities would make great problems to form the basis of argumentation in their classroom.

Since prior research on PD has shown that ongoing PD is better than one time PD, it was arranged for the teachers to have a weekly 1-hour PD session with their mentor to discuss how things were going, to talk about issues and challenges around getting kids to talk about math and for teachers to share what was working well. As experienced teachers, using argumentation already in their classrooms, the mentors provided suggestions from their own experience to get students engaged in discussions, to encourage resistant speakers, and to set clear expectations for classroom behavior to promote a safe environment for this kind of discussion.

Teachers were invited to participate in a Ning, a social network tool for collaboration, where they could access pictures of student work taken in different Jumpstart classrooms, videos from the PD, recorded overviews of each day's activities, blog about their experiences, and connect with other teachers. This was a new experience for many of the teachers. For Jumpstart 2009, PbWorks, an online collaborative workspace for document sharing, was created, but only a few teachers ended up using it. It was not clear how useful this tool was for the teachers. Jing, a screen casting software, was used to record audio and video descriptions of the curriculum for the day providing brief information about student activities shown on the screen. These videos were limited to 5 minutes for each day. The Jing video clips were available for teachers to access on the Ning and the links to the Jing daily overview videos were e-mailed to the teacher weekly. This research study did not include plans to gather data on usage of this technology. However, the district will gather data from feedback from teachers prior to Jumpstart 2011 to determine if either of these technology tools was beneficial to the teachers. There continues to be a need to think about how to provide ongoing PD so that teachers will engage more regularly in learning and sharing through an online community.

During the last weekly 1-hour PD sessions led by the mentors, the teacher posttest PCK assessment was given to participating teachers. The student pre and post-assessments were collected on the last day of the program by the UT grad student and delivered to the researcher for analysis once the data were de-identified.

Data Collection Measures

Student achievement. To measure student achievement, a pretest was given on the first day of the Jumpstart program to the students. It is part of the America's Choice Navigator (2009) curriculum entitled Positive and Negative Numbers. A posttest from the curriculum was given on the 14th day of the program to allow for make-up tests on the last day of the program. These same pre and posttests have been used with this Jumpstart program for the past two years. There is no information from the company on reliability and validity of the measures. They are newly developed. Their older navigator curriculum modules have been used in research and have been validated. However, that information is not available for this module. The student results are used to compare performance for students in Jumpstart 2009 to students in Jumpstart 2010 using the total percentage correct out of 25 items on the pre and post-assessment. Appendix F provides a sample of the assessment given to the students. For students with consent to participate in the study, their assessments were analyzed in more detail for accuracy on each set of problems based on operation type and for strategy use demonstrated by how students wrote their solution to the problems. Chapter 5 presents the findings from this more detailed review of the assessments of the participating students.

Teacher demographics. Teachers who have agreed to participate in the research study completed a brief demographic survey provided in Appendix E. Currently, only the number of years of teaching experience is used in the analysis due to the negative association of this variable with student performance at the end of Jumpstart 2009. The rest of the demographics are provided descriptively to inform the reader of the teacher sample characteristics. Due to the small sample size, it was deemed not appropriate to use any other demographics in the analysis.

Teacher PCK. To measure the impact of the PD on teacher PCK, an assessment was designed to use at the beginning of the PD and to administer towards the end of the 15-day Jumpstart program at the last meeting with the teachers. There were three sources based on a literature review used to develop the assessment: results from the student pilot study described in chapter 3, a pre-service teacher's guide entitled Fostering Children's Mathematical Power: An Investigative Approach to K-8 Mathematics Instruction (Baroody & Coslick, 1998), and research by Turnuklu and Yesildere (2007) with pre-service teachers in Turkey about their mathematical understanding of integer operations

Due to the results of the integer pilot study, the researcher wanted to include a question about real world and non-math domain applications of integers. The students amazed the researcher by the extensive diverse list of applications they were able to provide, so she wanted to see if the Jumpstart 2010 teachers also had knowledge of the extensive applications of integers. Subtraction and multiplication of integers were two areas where students in the pilot study had misconceptions. Turnuklu and Yesildere (2007) found similar misconceptions in the pre-service teachers in their study. There were pre-service teachers with misconceptions about addition of integers ($^{-}3 +5$) and subtraction of integers (3 –5). Barrody and Coslick (1998) used the following two questions for pre-service teachers to help them think about how to answer student questions:

1. A student teacher reads the expression $^{-}6 + (^{+}7)$ as “minus six and plus seven” and $6 – (^{+}7)$ as “six minus plus seven.” Is this good educational practice? Why or why not?
2. What is the answer to $5 – (^{-}8) = ?$ How would you explain to a student the answer you got? What is the answer to $-5 \times (^{-}8) = ?$ Why does it have the sign it does? (Barrody and Coslick, 1998, p. 8.22)

These three sources of problems allowed the researcher to create an assessment with eight questions (see Appendix D) to give to the teachers before PD and then near the end of the program to assess the change in their content and pedagogical knowledge.

In order to determine how to score the teacher PCK assessment, the researcher sent out an e-mail and the original version of the assessment to 30 science and mathematics education graduate students and mathematics teachers as representatives of the range of responses the researcher might get back from experts to novices in math content and from experts to novices in teaching expertise. Thirteen people volunteered to complete the survey and provide the researcher with their years of experience and the time it took them to complete the survey.

Seven of the volunteers were graduate students and the other six were teachers she had previously taught with in district. Five of the graduate students had no prior teaching experience. The other eight volunteers had teaching experience that ranged from 1 year to 14 years. These 13 volunteers reported taking 20 to 30 minutes to complete the 9-item survey (one question was omitted following this pilot so only eight items remained for the assessment used in this study). Based on patterns in their responses, the researcher created a rubric of descriptions of understanding to rate a participant's response on each item as a 0, 1, or 2. The scoring rubric is provided in Appendix G. Using this scoring rubric, the volunteers' responses were between 44% and 89%, which left room for study participants to exceed their scores or earn scores beneath their scores.

Constructivist Learning Environment Survey (CLES). This short 10-minute survey developed by Taylor and Fraser (1994) at Curtin University of Technology, was intended specifically for science classrooms, and includes five scales: "Learning about the World (Personal Relevance)," "Learning about Science (Uncertainty)," "Learning to Speak Out (Critical Voice)," "Learning to Learn (Shared Control)," and "Learning to Communicate (Student Negotiation)." The survey consists of 25 questions with five possible answers to each question: *almost never, seldom, sometimes, often, almost always*. Even though the survey was designed for science classrooms, the questions and responses are applicable to mathematics. The CLES survey was used in 1996 by the Dallas Public Schools as part of a systemic reform in mathematics and science. It was given to 1,600 students in 120 high school classrooms. All the values of reliability exceeded 0.80 except for the value for uncertainty (Dryden & Fraser, 1996). It was also given to 494 thirteen-year old students in the Australian component of the Third International Mathematics and Science Study (TIMSS) in 1994 resulting in reliability of above 0.80 in Personal Relevance, Critical Voice, Shared Control, and Student Negotiation, with Uncertainty still being lower at 0.72 using Cronbach's alpha. The factor analysis conducted by both of these studies confirmed the factor loading for the six items in each of the five categories (Taylor, Fraser,& Fisher, 1997). Therefore the CLES was used to measure each of these factors of a Constructivist Learning Environment for students and teachers in the Jumpstart program. A comparison of teacher responses to student responses was made. An analysis was conducted of any mediating role the level of constructivist learning environment may play in student growth from pretest to posttest. The results of these analyses are provided in chapter 5.

Student focus groups. The graduate student recorded a discussion with students about the curriculum activities during a focus group session. A series of prompts were provided to stimulate the conversation. The prompts are provided in Appendix H. This information is not part of the main analysis for the study. However, it provides background on the thinking of some of the students in the program.

Data Analysis

Question 1: What are the general patterns of teacher content and pedagogical knowledge of integers based on responses to a pre and post assessment? For this analysis, descriptive statistics will be used to describe the general patterns of teacher PCK based on coded responses to the pre and post-assessment items using means for each rating for each item and the means and standard deviations of the overall total PCK scores for the pretest and the posttest.

Question 2: To what extent does professional development impact teacher content and pedagogical knowledge as measured by growth between pre and post assessment? This question is answered with a paired samples t-test comparing the teachers who completed both the pretest and the posttest to determine statistically significant differences. This will help explain what areas of PCK were effected the most by the PD and implementation of the curriculum in the classroom.

Question 3: Is there a statistically significant difference between students' pre and post-test score for Jumpstart 2010 compared to the growth made by students in Jumpstart 2009? Since the school district refused to allow an experimental design with an intervention group and a control group, the third research question is answered by comparing the performance of Jumpstart 2010 students to the performance of the comparison group of students who participated in Jumpstart 2009 using an independent samples t-test while adjusting for clustering of students within classrooms using Hierarchical Linear Modeling to determine the adjusted standard error, t-statistic, and p-value. The HLM model is provided as follows:

Level-1 Model

$$Y_{ij} = \beta_0 + r_{ij}$$

Level-2 Model

$$\beta_0 = \gamma_{00} + \gamma_{10} \text{ (YEAR)} + u_{0j}$$

Y_{ij} was used to represent each outcome measure (pretest and posttest) and the growth of students between pre- and posttest. This model was run three times, once using the outcome variable for pretest, once using the outcome variable for posttest, and finally using the outcome variable of growth. A two level model was used to account for the nesting of students (i)within classes (*j*). Year represented allowing the model to account for assignment of students to Year 2009 or Year 2010. The regression coefficients β_{0j} in the level one equation are class-specific, created from data at the student level, and differ across classrooms. The regression coefficient β_{0j} is treated as dependent variables at the class level (level two). The level-2 random effects, u_{0j} and u_{1j}, measured random variation in class posttest scores.

Question 4: Do differences in teacher content and pedagogical knowledge explain more of the variance in student performance (pretest/posttest) than years of teaching experience of a teacher? To answer these questions, Hierarchical Linear Modeling (HLM) was used to compare performance of students in Jumpstart 2009 to students in Jumpstart 2010 on an integer assessment and to determine the relationship between properties of the classroom (teacher content and pedagogical knowledge, teacher experience, and teacher beliefs) and properties of the students (changes in performance on assessment of understanding of integer operations and beliefs about learning). Because of the nested structure of the data, with students nested in classrooms, a hierarchical linear model is a statistical method the researcher used to analyze this relationship. Due to the presence of more than one random variable and a cross-level interaction between the variables at the classroom level and the variables at the student level, a multilevel analysis is recommended by the research community (Raudenbush & Bryk, 2002).

A two-level HLM model was used with student math scores as the level one outcome variable (a within-class model) and individual class variables, such as mean math score, as the level two outcome variables (a between-class model). Specifically, each student, *i*, in classroom *j*, where, $i\ ,\ j\ \mathcal{E}\ Z^+$ can be represented by the following equation $y_{ij} = ß_{0j} + ß_{1j}(x_{ij}) + r_{ij}$. This equation states that the posttest score of student *i* in class *j* can be decomposed into the average posttest score in class *j* ($ß_{0j}$), an adjustment based on the product of the pretest score (x_{ij}) and the class pre/posttest performance slope ($ß_{1j}$) and a randomly varying error term (r_{ij})~ $N(0,\sigma^2)$. Group mean-centering will be used first for the level one pretest score. Then if the slope variance is not significant, grand mean centering will be used with the slope ($ß_{1j}$) as a fixed effect at level two with the residual term at zero.

At the second level, each classroom will have a mean posttest score ($ß_{0j}$) and a pre/posttest performance slope ($ß_{1j}$) as on outcome. The analysis began with the random coefficient model. Then classroom-level predictors were added into the level two model to determine whether they explain the variance in mean class scores and pre/posttest performance slopes: teacher content and pedagogical knowledge of integer variables based on the results of the pre and posttest and the teacher demographic of years of teaching experience.

Because the relationships between variables measuring student learning are at least partially dependent on the larger context (the classroom) in which learning occurs, the expectation is that teacher content and pedagogical knowledge[8] can partially explain the variance across classes, although years of experience of a teacher might also explain the variance across classes. An investigation was conducted to determine how each of these teacher variables explain the variance across classes and whether content and pedagogical knowledge would outweigh experience in explaining the variance across classes. What follows are the unconditional model and the potential conditional model. Chapter 5 presents the progression of changes in the models based on the data collected and the final outcomes.

Unconditional Model

Level-1 Model

$$Y_{ij} = \beta_0 + \beta_1\text{*(Student Pretest)} + r_{ij}$$

Level-2 Model

$$\beta_0 = \gamma_{00} + u_{0j}$$

$$\beta_1 = \gamma_{10} + u_{1j}$$

If the slope variance is found to be not significant then the unconditional model with slope as fixed effect at level 2 will be used as follows:

[8] The CLES teacher survey results were not used as a covariate in the HLM model for two reasons. First, the theory of change for this study focuses on changes in teacher PCK following PD and the associated change in student understanding of integer operations; therefore, teacher PCK is the focus of the HLM analysis. Second, CLES was used to measure if students reported using classroom practices similar to what they experienced in PD (use of mathematical conversations and argumentation); however, the CLES represents teacher self reports not actual evidence of use of these practices. Further analysis of the relationship between teacher CLES results and student performance will be considered for a future study.

Unconditional Model with Slope as Fixed Effect at Level 2

Level-1 Model

$Y_{ij} = \beta_0 + \beta_1$*(Student Pretest) + r_{ij}

Level-2 Model

$\beta_0 = \gamma_{00} + u_{0j}$

$\beta_1 = \gamma_{10}$

Before entering predictor variables into the fully conditional HLM model the researcher determined whether the variation in mean achievement (β_{0j}) and achievement slopes (β_{1j}) were significantly different than 0 to determine whether the null hypothesis $\tau_{00} = 0$ and $\tau_{11} = 0$ should be rejected and use a model with level-2 predictors. Then level 2 variables were entered into the fully conditional model to determine if any were statistically significant and the extent to which they explained the variance in mean achievement.

Fully Conditional Model

Level-1 Model

$Y_{ij} = \beta_0 + \beta_1$*(Pretest) + r_{ij}

Level-2 Model

$\beta_0 = \gamma_{00} + \gamma_{01}$*(Teacher Experience) + γ_{02}*(Teacher PCK Pretest) + γ_{03}*(Teacher PCK Posttest) + u_{0j}

$\beta_1 = \gamma_{10}$

All predictor variables were grand mean centered, because the slope variance was found to not be significant. The preceding conditional model is written with slope as a fixed effect which was conducted because the slope variance in the unconditional model was found to not be significant.

Question 5: What is the relationship between teachers' responses to the Constructivist Learning Environment Survey (CLES) and the student responses to the CLES? A comparison of the student and teacher response to the CLES subscales and composite score will be conducted using an analysis that compares the teacher response to the box plot of the student responses for their class to determine if teachers experience higher, lower, or similar levels as their students for each subscale and for the total CLES score. The results of this analysis and the results for the other four analyses are presented in Chapter Five.

CHAPTER FIVE: FINDINGS

In this chapter, the researcher presents the results of this study related to each research question as well as additional data collected from student focus groups to provide additional background to the study. The first set of analyses provides a description of the participating teachers based on demographics and the assessment of PCK. The second set of analyses describes differences between students' performance on the integer assessment for Jumpstart 2010 compared to the students' performance on this same assessment during Jumpstart 2009. The third set of analyses looks at the relationship between teacher characteristics and student performance on the integer assessment to see if any of the variance in student performance can be explained by teacher characteristics taking into consideration the nesting of students within classrooms. The fourth set of analyses looks at assessment and survey data for a subsample of students with parent consent and student assent to better understand changes in their understanding during the Jumpstart program. The final analysis provides some background information based on focus group interviews with students that occurred towards the end of the Jumpstart program.

Analysis of Differences Between Teachers

Research Question 1

What are the general patterns of teacher content and pedagogical knowledge of integers based on responses to a pre and post assessment?

The pretests and posttests of teacher PCK were scored using the rubric provided in Appendix G. There were three teachers who arrived late to the PD and were unable to take the pretest. All teachers completed the posttest. One of the teachers did not attend the PD and was replaced by a substitute. Since neither of them consented to participate in the study, their data are not included in this analysis. The performance data of their students are used only for analyses that do not include teacher level variables.

As described in chapter 3, the teacher PCK Assessment was designed to measure each teacher's understanding of the real world and non-mathematical domain applications of integers, of the conceptual meaning of integer operations beyond memorized rules for the operations, and of common student misconceptions related to integers and integer operations. Table 20 provides a summary of the teachers' performance on the assessment prior to the PD and at the end of the Jumpstart program.

Table 20 Pedagogical Content Knowledge Assessment Performance (Percent of 100)

Variable	Minimum	Maximum	Mean	Standard Deviation
Pretest (*n* = 18)	7	93	46	21
Posttest (*n* = 21)	29	93	62	20

Note. Source: Researcher's analysis of Teacher Assessments of PCK.

The results of the teacher pretest and posttest performance are summarized in Table 21 by item to describe the weaknesses in understanding at baseline and the weaknesses that still persist at posttest. At pretest, a majority of teachers (77%) could not provide a rationale for why multiplication of two negative numbers results in a positive number. At the posttest this number was reduced to 48% of teachers, which is still a large percent of teachers who have more of a rule-based understanding of multiplication of integers. At pretest, a majority of teachers (61%) believed that 3 –5 was the same as 3+ (-5). At the posttest only 24% of teachers still believed they were the same. Even though both expressions result in the same answer when simplified, there is different meaning in the operation of addition and subtraction. These two problems exemplify two common student misconceptions that resulted in errors in response during the pilot study. This improvement in teacher understanding of these concepts and student misconception should be related to student understanding.

There were also improvements in teachers' ability to identify both real world and domain applications of integers, from 17% at pretest to 38% at posttest. This was not a focus of the PD; however, the curriculum exposed teachers and students to real world and domain applications which may have contributed to this change in understanding. As to be expected more teachers reported having experience with argumentation in their class following PD and implementation of the curriculum in their classrooms. However, 24% admitted to not experiencing argumentation in their class. Within their written feedback these five teachers explained that their students were resistant to talking about mathematics in class or that students were not willing to participate in argumentation.

Table 21 Item Analysis for Pedagogical Content Knowledge Assessments

Percent of Teachers		Pre-test Score Distribution (Points per Question) (n = 18 teachers)			Posttest Score Distribution (Points per Question) (n = 21 teachers)		
Question	Rating	**0**	**1**	**2**	**0**	**1**	**2**
How would you explain the solution of 5 – (-8)?		50	33	17	43	10	47
Given -5 x (-8). Why does the answer have the sign it does?		72	17	11	48	10	42
(-6) + (+7) and 6 – (+7) read incorrectly		22	28	50	0	24	76
4 – 7 = 3, what is the misconception and what is a teaching strategy		6	44	50	5	33	62
Is 3 –5 the same as 3 + (-5)? Explain		61	28	11	24	62	14
Describe prior experience with argumentation in class		33	28	39	24	29	48
Real world and domain applications of integers		11	72	17	14	48	38
Total Score (14 possible points)		Mean: 46 Standard Deviation: 21			Mean: 62 Standard Deviation: 20		

Note. Source: Researcher's analysis of Teacher Assessments of PCK.

Research Question 2

To what extent does professional development impact teacher content and pedagogical knowledge as measured by growth between pre and post assessment?

When the pretest and posttest responses of the 18 teachers were compared based on the overall score[9], the growth made from pretest to posttest in teachers' PCK was statistically significant (t = 3.29, p < .01). The question that received the most statistically significant change was when teachers were shown a statement made by a student teacher who misread the operations and signs in two problems. Teachers on the

[9] A paired samples t-test was used as a test of statistical significance using SPSS statistical software and confirmed by a parallel analysis with SAS software.

posttest were more aware of the importance of differentiating between an operation and a sign when speaking to students about expressions with positive and negative numbers. This significant change for Research Question 3 is shown in Table 22.

Table 22 Comparison of Pretest and Posttest Results by Question

Question (N = 18 teachers)	Pretest Mean (SD)	Posttest Mean (SD)	Difference (SE)	t	p-value
Q1. 5 – (-8)	.67 (.77)	1.11 (.96)	.44 (.32)	1.41	.18
Q2. -5 x (-8)	.39 (.70)	1.00 (.97)	.61 (.26)	2.37	.03*
Q3. (-6) + (+7), 6 – (+7)	1.28 (.83)	1.83 (.38)	.56 (.17)	3.34	$p < .01$
Q4. 4 – 7 = 3	1.44 (.62)	1.61 (.61)	.17 (.12)	1.37	.19
Q5. 3 – 5, 3 + (-5)	.50 (.71)	.89 (.68)	.39 (.20)	1.94	.07
Q6. Prior Argumentation	1.06 (.87)	1.22 (.88)	.17 (.20)	.83	.42
Q7. Applications	1.06 (.54)	1.33 (.59)	.28 (.11)	2.55	.02*

Note. (a) These values represent the average raw score out of 2 points earned by teachers on the pretest and the posttest for each question. Teachers could earn 0, 1, or 2 points per question. The total questions were worth 14 points. (b) *Statistically significant difference at the $p < .05$ level. (c) See Table 25 for question detail.

There was also statistically significant growth (t =2.37, p <.05) in teachers' ability to explain the sign of the answer to the problem -5 x (-8). Teachers also made statistically significant growth (t =2.55, p < .05) in knowing real world and domain examples of applications for integers. What is not clear is whether the curriculum and PD introduced new applications to them, if they learned additional applications on their own, or if they just remembered more at the posttest. Due to the significance of this growth, the teacher PCK score was used in the HLM model to determine whether differences in teacher PCK accounts for differences in student performance from pre to posttest.

There were also many real world and domain applications of integers that teachers listed on their PCK assessments. These responses are provided in Appendix I to show the diversity of responses and the wide range of applications in mathematics, other domains, and in everyday life. These examples are a great resource for problem creation for students to think about.

Comparison of Student Achievement (Jumpstart 2009 vs. 2010)

Research Question 3

Is there a statistically significant difference between the growth between students' pretest and posttest score for Jumpstart 2010 compared to the growth made by students in Jumpstart 2009?

To compare student performance between the two years of the Jumpstart program, a histogram of student performance from pretest to posttest for each year was used to look at patterns of change between the two years. For Jumpstart 2009, some of the students lost ground over the 3-week program. In 2010, the Jumpstart students started on average about 10% lower than the Jumpstart 2009 students. However, they made almost twice as much growth and very few students lost ground. Histograms are provided to show differences in change in performance for Jumpstart 2009 (See Figure 5 an 6) and for Jumpstart 2010 (See Figures 7 and 8).

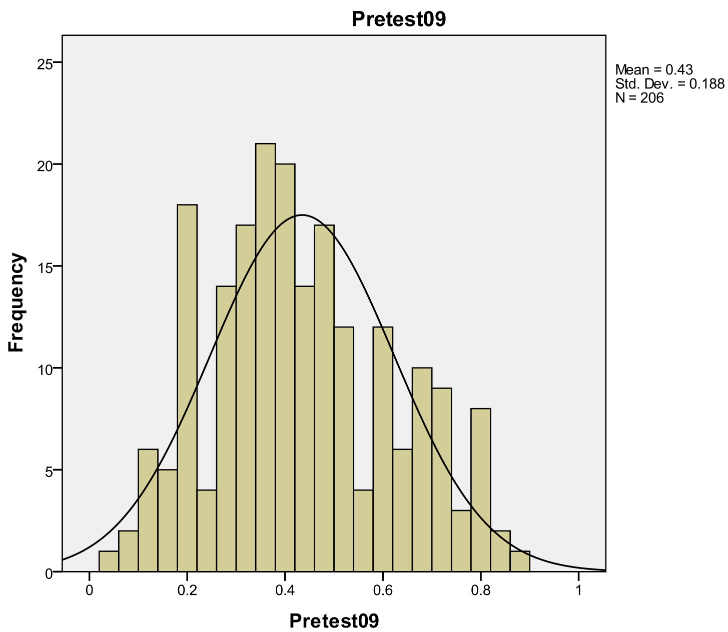


Figure 5   Jumpstart 2009 pretest student performance.

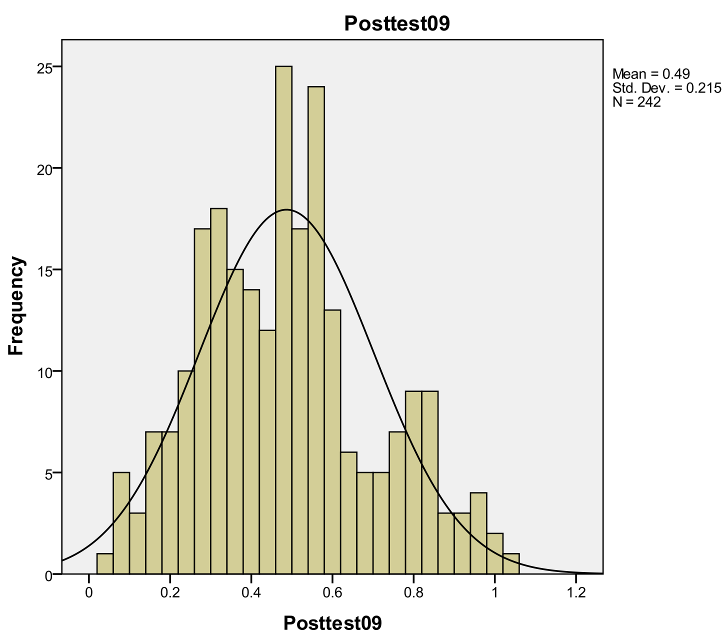


Figure 6   Jumpstart 2009 posttest student performance.

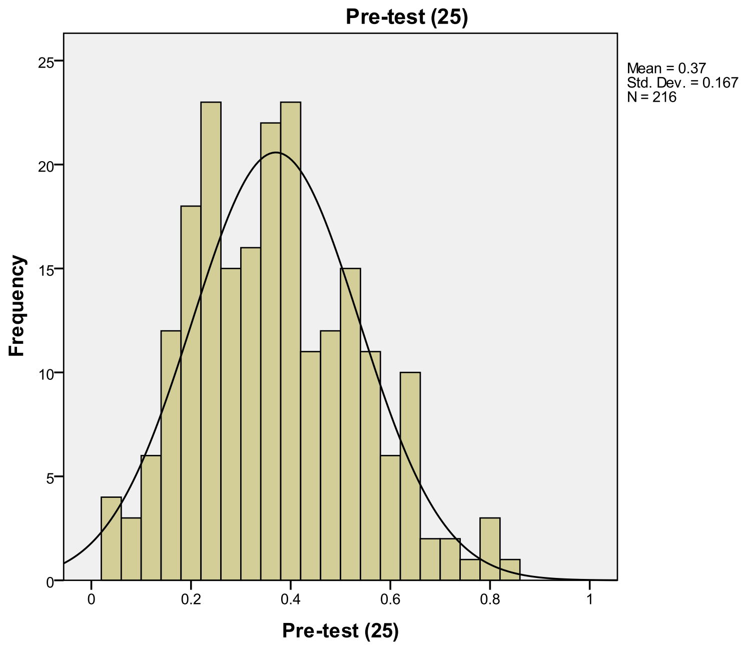


Figure 7   Jumpstart 2010 pretest student performance.

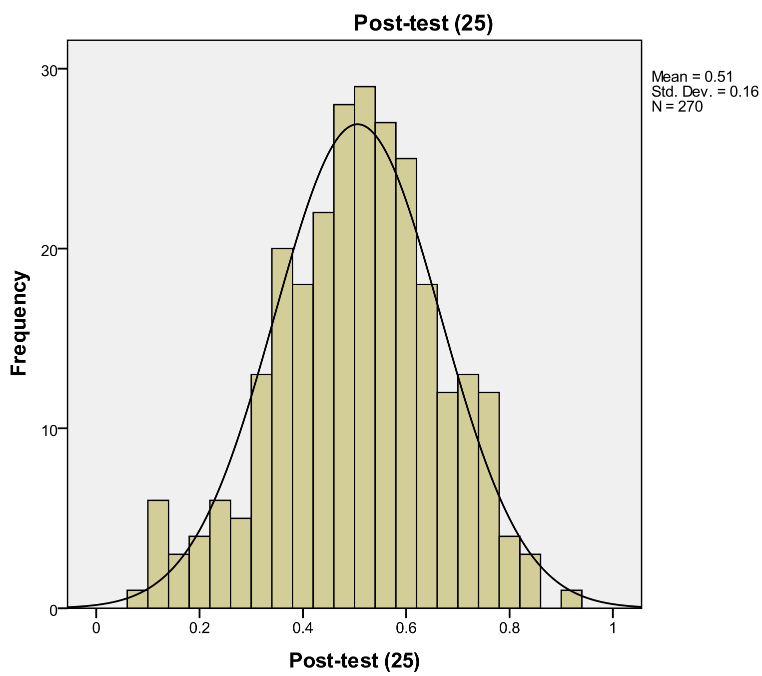


Figure 8   Jumpstart 2010 posttest student performance.

An independent samples *t*-test was calculated to determine if there was a statistically significant difference between the growth made in student performance for Jumpstart 2010 as compared to Jumpstart 2009. Then Hierarchical Linear Modeling was used to account for the nesting of students within classrooms to adjust the standard error and resulting *t*-statistic and *p*-values. The HLM model is provided as follows:

Level-1 Model

$$Y_{ij} = \beta_0 + \beta_1 * (\text{YEAR}) + r_{ij}$$

Level-2 Model

$$\beta_0 = \gamma_{00} + u_{0j}$$

$$\beta_1 = \gamma_{10} + u_{1j}$$

Y_{ij} was used to represent each outcome measure (pretest and posttest) and the growth of students between pretest and posttest. A two-level model was used to account for the nesting of students (*i*)within classes (*j*). Year represented allowing the model to account for assignment of students to Year 2009 or Year 2010. The regression coefficients β_{0j} and β_{1j} in the level one equation are class-specific, created from data at the student level, and differ across classrooms. The regression coefficients β_{0j} and β_{1j} are treated as dependent variables at the class level (level two). The level-two random effects, u_{0j} and u_{1j}, measured random variation in class posttest scores.

Table 23 provides the outcome of the comparison. The student performance at pretesting for 2010 was lower (but statistically significant) than students in 2009 by 7% lower on average across the sample. Student performance at posttesting for 2010 was slightly higher than students in 2009, but not statistically significant. The student growth made from pretest to posttest for Jumpstart 2010 was statistically significantly higher than the growth made in 2009 (t = 2.19, p = .03). Therefore, there was statistically significant difference between the growth made this year during Jumpstart 2010 as compared to Jumpstart 2009.

Table 23 Comparison of the Performance of Students in Jumpstart 2009 vs. 2010

Percent Correct out of 100	2010 Mean (n=178) (SD)	2009 Mean (n = 177) (SD)	Difference (SE)	t	p-value
Pre-test	37 (17)	43 (19)	-7 (4)	-1.77	.08
Posttest	51 (15)	49 (21)	2 (4)	.41	.68
Growth	14 (17)	06 (19)	8 (3)	2.20	.03*

*Not*e: Source: De-identified performance data for students participating in Jumpstart 2009 and 2010 analyzed using an independent samples t-test to determine the means and standard deviations, and then analyzed with HLM to correct t-statistics, standard error and p-values for nesting within classrooms.
*Statistically significant at the p < .05 level.

Hierarchical Linear Model Analysis

Research Question 4:

Do differences in teacher content and pedagogical knowledge explain more of the variance in student performance (pre/post test) than years of teaching experience of the teacher?

Hierarchical linear modeling was conducted to determine whether or not there was a relationship between teacher PCK and student performance on the posttest.

Specifically, a two-level model was tested; the first level consisted of student variables while the second level consisted of teacher or classroom variables.

Steps in Model Testing

The first step in any HLM procedure involves testing the fully unconditional model. This model does not include any predictors at the first and second levels. Further, only the level-one intercept (i.e., the mean of the dependent variable) is modeled at the second level. The objective of testing the fully unconditional model is to determine whether there is significant variance in the dependent variable (i.e., student posttest performance). If there is significant variance in the dependent variable, then the second step is taken.

The second step involves testing the unconditional model with a first-level predictor. In addition, both the level-one intercept and slope are modeled at the second level. The objective of testing this unconditional model is to determine whether the intercept and the slope vary significantly between students. If either or both the intercept and the slope vary significantly between students, then model testing proceeds further.

In the third step, the conditional model is tested. In this step, second-level predictors are added into the model so that the variance in the intercept and/or slope can be explained. The second-level variables predicted to account for variance in the intercept and slope of posttest student performance were teaching experience (in years), teacher pretest PCK score, and teacher posttest PCK score.

Results for the Unconditional Model Tests

The fully unconditional model was as follows:

Level-1 Model

$$Y_{ij} = \beta_0 + r_{ij}$$

Level-2 Model

$$\beta_0 = \gamma_{00} + u_{0j}$$

There was significant variance in posttest score across classes (χ^2 (17) = 28.33, p = .000). Therefore, an unconditional model with student pretest score (group mean centered) was included as a level-one predictor. In addition, both the level-one intercept and slope were modeled at the second level. The unconditional model was as follows:

Level-1 Model

$$Y_{ij} = \beta_0 + \beta_1\text{*(Student Pretest)} + r_{ij}$$

Level-2 Model

$$\beta_0 = \gamma_{00} + u_{0j}$$

$$\beta_1 = \gamma_{10} + u_{1j}$$

Although there was significant variance in the intercept (χ^2 (16) = 52.40, p = .000), the slope variance was not significant (χ^2 (16) = 25.27, p = .065). Thus, the level-one slope was fixed in succeeding analyses and the student pretest score was grand mean centered[10]. Since there was still significant variance in posttest student scores even after accounting for nesting in classrooms and controlling for student pretest scores, the predictors of interest were added into the second-level model to determine whether these predictors could significantly explain the variance in posttest student scores.

[10] For completeness, group pretest was used as a predictor for the level one intercept to test for contextual effects associated with class pretest. However, it was not significant, so it was removed from the model.

Results for the Conditional Model

The conditional model was as follows:

Level-1 Model

$$Y_{ij} = \beta_0 + \beta_1\text{*(Pretest)} + r_{ij}$$

Level-2 Model

$$\beta_0 = \gamma_{00} + \gamma_{01}\text{*(Teacher Experience)} + \gamma_{02}\text{*(Teacher PCK Pretest)} +$$
$$\gamma_{03}\text{*(Teacher PCK Posttest)} + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

The findings for this conditional model are summarized in Table 24. The findings reveal that, after controlling for prior student and teacher knowledge as well as teacher experience, teacher's PCK (PCK) significantly predicted student posttest performance (t (14) = 2.37, p = .033). Thus, as teacher's post-intervention PCK increased by one point, there was a 0.22 increase in student math posttest performance. This positive relationship between teacher posttest PCK and posttest student math achievement suggests that when teachers have the necessary PCK of integers and integer operations, students' overall learning of integer concepts and procedures for solving integer operation problems improve.

When comparing the conditional model to the unconditional model, the addition of the teacher experience, teacher pretest knowledge and teacher posttest knowledge covariates explained 27% of the variance (i.e., 33.80-24.58/33.80) after adjusting for differences in student pretest. Sensitivity analyses were conducted to determine the robustness of the model if one or more of the predictors at level two were not included in the model. Teacher posttest PCK remained the only significant predictor. This fully conditional model is reported because it is a more robust estimate of the relationship between teacher PCK and student post-test scores.

Table 24 Two-level Model Estimates of the Impact of Teacher Pedagogical Knowledge on Student Performance on the Positive and Negative Numbers Posttest

Parameter	**Unconditional Model**				**Conditional Model**			
	Estimate	*SE*	*t*	*p*	Estimate	*SE*	*t*	*p*
Fixed effects								
Intercept	50.79	1.73	29.28	.000	50.49	1.59	31.70	.001
Student Pretest	.44	.06	6.78	.000	.45	.06	7.03	.000
Mean Teacher Experience	—	—	—	—	.04	.23	.17	.866
Mean Teacher PCK Pretest	—	—	—	—	.00	.09	-.04	.968
Mean Teacher PCK Posttest	—	—	—	—	.22	.09	2.37	.033
Random effects	Variance	*df*	χ^2	*p*	Variance	*df*	χ^2	*p*
Intercept	33.80	17	53.74	.000	24.58	14	36.26	.001
Level 1 variance	135.02	—	—	—	135.31	—	—	—

Note. Total number of students was 146. Total number of teachers was 18. All predictor variables were grand mean centered.

The results suggest that paying attention to teacher PCK as it relates to integer concepts and operations was an important contribution to the Jumpstart program this year. By participating in PD, teacher PCK was changed. Increases in teacher PCK were associated with statistically significant gains in student achievement between the pre and posttest controlling for baseline teacher pretest PCK, student knowledge at pretest, and years teaching experience. To better understand the impact of the PD on class instruction and the implementation of strategies for solving integer operations beyond just a rule based understanding, the next analysis looks deeper into a subset of 102 students who returned parent consent forms and signed student assent forms for their responses to be analyzed in more detail to better understand what students experienced in their classrooms during Jumpstart and changes in their understanding of positive and negative numbers.

Comparison of Teacher and Student Constructivist Learning Environment Survey

The CLES was given to measure to what extent teachers allowed students opportunities to take ownership of the learning by talking about math with other students, sharing their thinking with the class, and being involved in instructional decisions. Time and resources did not allow the opportunity to observe classes to determine the level of implementation of the strategies discussed in the PD related to getting students to talk about mathematics through argumentation. The CLES is an indirect measurement of whether or not these kinds of constructivist behaviors were implemented according to teacher self-report. A similar survey was given to students. That analysis is provided later in this chapter. Table 25 provides the results of the teacher CLES survey.

Table 25 Constructivist Learning Environment Survey Responses (N = 21)

Variable	Description*	Min.	Max.	Mean (*SD*)
Knowledge of Content				
Personal Relevance	Extent to which school mathematics is relevant to students' everyday out-of-school experiences.	0.50	0.80	0.62 (0.08)
Uncertainty	Extent to which opportunities are provided for students to experience that mathematical knowledge is evolving and culturally and socially determined.	0.27	0.93	0.50 (0.18)
Knowledge of Students				
Critical Voice	Extent to which students/teachers feel that it is legitimate and beneficial for students to question the teachers' pedagogical plans and methods.	0.63	1.00	0.84 (0.12)
Knowledge of Pedagogy				
Shared Control	Extent to which students have opportunities to explain and justify their ideas, and to test the viability of their own and other students' ideas.	0.33	1.00	0.65 (0.23)
Negotiation	Extent to which students share with the teacher control for the design and management of learning activities, assessment criteria, and social norms of the classroom.	0.40	1.00	0.79 (0.17)
Total		0.57	0.91	0.68 (0.08)

Note. Source: Analysis of Teacher CLES Survey Responses and description of variables.

* Taylor, Fraser, & White (1994)

The CLES provides insight into the Jumpstart teachers' perceptions of their classroom environment. The low ratings of uncertainty may be related to the curriculum for the Jumpstart program which focused more on mathematical concepts and skills rather than connecting it to a larger sociological context or a relevant cultural application. This may also explain the low ratings for personal relevance. However, low relevance could also mean that teachers are not as aware of the relevance of integer operations. This data can inform the planning of the next Jumpstart program to improve the connection to student lives and experiences. The two areas most closely associated with the instructional strategy of argumentation explored during the PD are critical voice and negotiation. From the teachers' perspective, these two areas are highly rated so one would expect to see implementation of these behaviors in their classroom. However, teachers' perceptions are often more positive than their students (Johnson & McClure, 2004). What follows is a description of the student CLES responses and a comparison of student responses to teacher responses to determine whether students experienced similar levels of the subscales of the CLES survey.

Data for students who returned a signed parent consent form and who completed a student assent form were analyzed in more detail to look at responses to the CLES survey items and subscales, to compare their responses to the CLES survey to their teacher's responses to the CLES survey, to look at the strategies used on the pretest and the posttest to note any changes to better understand any additional relationship between participation in Jumpstart and change in student performance on the integer assessment from pretest to posttest.

There were 102 students who assented to participate in the study and who had signed parent consent to participate. They were recruited to help learn more about what students know about positive and negative numbers, what strategies they use to solve integer operations problems, and to look more closely at patterns of growth from their pretest to their posttests. Their testing data was analyzed more closely given their permission to provide additional background of this study. Since these students are a subset of the entire group of Jumpstart students (n = 341), the results of this analysis is not necessarily representative of the entire sample *of Jumpstart* students. However, information on strategy use may reveal changes in strategy use from pre to posttest which might inform future program and PD design. Descriptive statistics for this subsample of students are provided in Table 26.

Table 26 Description of the Subsample Average Scores on Outcome Measures (Percent of 100)

Variable	*N*	Mean	Standard Deviation	Minimum	Maximum
Achievement					
Pretest	53	39	15	4	76
Posttest	76	50	15	12	80
CLES					
Personal Relevance	69	63	13	37	100
Uncertainty	68	65	15	33	100
Critical Voice	71	68	15	37	100
Shared Control	68	63	17	20	100
Negotiation	69	69	15	30	100
CLES Total	63	66	11	45	97

Note. Source: Student assessments and CLES Survey with parent consent and student assent.
The 6 CLES subcategories consist of items that are rated from 1 to 5; there are five items for each subcategory resulting in 25 possible points per subcategory. The percent provided in the table is the total points out of 30 possible points, a subcategory received as rated by the student averaged across students. There are 150 total points possible.

Research Question 5

What is the relationship between teachers' responses to the Constructivist Learning Environment Survey (CLES) and the student responses to the CLES?

For each subscale of the CLES, a box plot was created to compare the student ratings to the teacher rating (see Appendix J). The box represents the range of student ratings from 0 to 1.0 (0 – 100%) which is followed by a small square dot that represents the teacher's rating. If the teacher's rating fell within the box of their student responses, their response was rated as "similar" to the responses of their students. If the teacher's rating fell above the box of their student's responses, their response was rated as "higher" than their students' ratings. If the teacher's rating fell below the box, then their response was rated as "lower" than their students' ratings. Next, the frequency of teachers within each rating: similar, higher, lower was calculated to determine the percent of teachers similar to students and the percent of teachers dissimilar to their students. Table 27 provides the summary information from that analysis.

Table 27 Comparison of Teacher and Student Responses to the Constructivist Learning Environment Survey

Variable (*N* = 17 teachers)	Similar %	Higher than Student Ratings %	Lower than Student Ratings %
Knowledge of Content			
Personal Relevance	35	30	**35**
Uncertainty	24	6	**70**
Knowledge of Students			
Critical Voice	29	**71**	0
Knowledge of Pedagogy			
Shared Control	35	**47**	18
Negotiation	23	**65**	12
Total	**59**	41	0

Note: Source: Analysis of teacher and student CLES survey responses.

The CLES Survey subscales are grouped into three main categories: Knowledge of Content, Knowledge of Students, and Knowledge of Pedagogy. In comparing teacher responses to student responses, a majority of the teachers' ratings were lower than student ratings for Knowledge of Content (Personal Relevance and Uncertainty). In general, teachers' ratings of Knowledge of Students (Critical Voice) were higher than students' ratings. Most teachers' ratings of Knowledge of Pedagogy (Shared Control and Negotiation) were also higher than students' ratings. This last category, Knowledge of Pedagogy, was the one the PD was most targeted towards improving in order to encourage teachers to allow students to take responsibility for their learning, to engage in argumentation, and to share their thinking with others. So the results of the survey show that the majority of teachers rate these kinds of classroom environment features highly, but their students were not as aware of them happening in the classroom.

Further research is needed to determine if more time or additional classroom structures improve students' experiences having a critical voice to express their opinions, having a say or control in decisions made about what they are learning, and negotiating their understanding by explaining their thinking to others and challenging ideas that are different. A limitation of this sub-sample analysis is that it represents only about 22% of the Jumpstart students and there was variability in the number of students per class with consent to participate in this survey. So these outcomes may not be representative of the entire sample.

Subsample Exploratory Analysis of Strategy Use

Although not one of the main research questions, in light of the research conducted in the pilot study related to student strategy use, data was collected for the consenting students on strategy use by students for the pre and posttest. First, the percent of each type of strategy use evident on the pretest and then on the posttest is presented for all students who agreed to participate in the study in Table 28. Then a summary of strategy use changes from pre and posttest for the 70 participating students who took both assessments is provided in Table 28.

Table 28 Percent of Students Using Different Strategies on Pretest and Posttest

Strategy	Pretest (n = 71)	Posttest (n = 101)
Change subtraction to addition	11 (32)	37 (49)
Number line (traditional method)	21 (41)	12 (32)
Vector Number line (Shows magnitude and direction)	1 (12)	5 (22)
Pyramid or Pieman (Memory strategy for multiplying integers)	2 (17)	5 (22)
Use of signs, counters, or sticks as objects to manipulate for integer operations	4 (20)	7 (25)

Note: Source: Item analysis of strategy use on pre and post assessment for a subset of participating students.

Table 29 Change in Percent of Students Using Different Strategies from Pretest to Posttest

Strategy (n = 70)	Pretest Mean (SD)	Posttest Mean (SD)	Difference (SE)	t-test	p-value
Change subtraction to addition	11 (32)	41 (50)	30 (7)	4.20	P <.001
Number line (traditional method)	21 (41)	13 (34)	-9 (6)	-1.43	.16
Vector Number line (Shows magnitude and direction)	1 (12)	4 (20)	3 (3)	1.00	.32
Pyramid or Pieman (Memory strategy for multiplying integers)	3 (17)	6 (23)	3 (3)	1.00	.32
Use of signs, counters, or sticks as objects to manipulate for integer operations	4 (20)	9 (28)	4 (3)	1.35	.18

Note. Source: Item analysis of strategy use on pre and post assessment for 70 participating Jumpstart students who took both the pre and posttests.

The results from Table 29 show that students in this subsample of participating students who took both the pre and posttest made a statistically significant increase in the use of the strategy referred to by many of the students and teachers as “Slash and Dash” where subtraction is changed into addition prior to solving the problem. To determine whether there was any evidence that this strategy use improved their accuracy, these 70 participating students with both pre and posttest results were compared on each subset of problems and the total score to determine areas of significant growth. The “slash and dash” strategy use was most related to improvements in solving integer subtraction problems. Table 30 compares the growth from pretest to posttest by problem type.

Table 30 Comparison of Students Pre and Post Assessment by Problem Type (n = 70)

Assessment Problem Type	Pretest Mean (*SD*)	Posttest Mean (*SD*)	Difference (SE)	*t*-test	*p*-value
Graphing Numbers on a Number Line	61 (38)	84 (23)	23 (5)	4.66	*p* <.001
Comparing Positive and Negative Numbers	66 (36)	75 (35)	09 (5)	2.03	*p* <.05
Addition of Integers	49 (41)	64 (40)	15 (5)	2.77	*p* <.01
Subtraction of Integers	22 (25)	38 (34)	16 (4)	3.81	*P* <.001
Multiplication and Division of Integers	38 (31)	47 (26)	9 (5)	1.77	.08
Variable Substitution with Integers to Simplify Expressions	9 (28)	11 (32)	3 (5)	.63	.53
Simplifying Algebra Expressions	3 (12)	9 (16)	6 (2)	2.54	*P* <.05
Solving Inequalities with Integer Solutions	0	0	0	0	0
Total Score (20 Items most related to curriculum)	46 (19)	61 (18)	15 (2)	6.03	*p* <.001
Total Score (Complete 25 item test)	38 (15)	51 (15)	13 (2)	6.19	*p* <.001

Note. Source: America's Choice Navigator, *Positive and Negative Numbers*, Pre and Post-Assessment.

Students made statistically significant growth ($p < .05$) on all subsets of problems on the test except multiplication and division of integers and variable substitution with integers to simplify expressions. They also made statistically significant growth ($p < .001$) on the entire assessment (25 items) as well as the first 20 items which were directly related to the Jumpstart curriculum. The correlations between the total score, subsets of problems, and strategy use were analyzed. Statistically significant correlations are presented in the following table. Appendix K and L provides additional information on other correlations.

Table 31 Statistically Significant Correlations between Subtest Scores and Strategy Use

Variable 1	Variable 2	Correlation	Level of Statistical Significance
PRETEST			
Comparing Numbers	Use of Objects	-.24	.04 *
Simplifying Expressions	Change subtraction to addition	.29	.02 *
Simplifying Expressions	Vector Number Line	.31	$p < .001$
Simplifying Expressions	Pieman or Pyramid Memory Strategy	.45	$p < .0001$
POST TEST			
Addition of Integers	Change subtraction to addition	.44	$p < .001$
Subtraction of Integers	Change subtraction to addition	.39	$p < .01$
Total Score	Change subtraction to addition	.38	$p < .01$

Note. * Statistically significant at the $p < .05$ level.

The total test score for the pretest was not statistically significantly correlated with any strategy use by these 70 participating students. Due to the small number of students who are considered as participants in each class and the variance in number of participating students in each class, a paired samples *t*-test was more appropriate than using HLM to account for nesting within classrooms. If a larger number of students had returned consent forms and completed assessments, an HLM analysis may have been more appropriate for a comparison. Table 32 provides the distribution of participating students with pre and posttest results by classroom which shows that all classes are not represented, so the outcomes needed to be interpreted with caution.

Table 32 Distribution of the 70 Participating Students with Pretest and Posttest Results by Classroom

Classroom	Students
Teacher A	0
Teacher B	8
Teacher C	0
Teacher D	1
Teacher E	1
Teacher F	2
Teacher G	0
Teacher H	2
Teacher I	1
Teacher J	0
Teacher K	4
Teacher L	3
Teacher M	2
Teacher N	6
Teacher O	4
Teacher P	4
Teacher Q	4
Teacher R	9
Teacher S	3
Teacher T	6
Teacher U	3
Teacher V (nonparticipating)	5

Note. These students completed both the pre and posttest, submitted a parent consent form and a student assent form. Teachers A-U consented to participate. Teacher V was a substitute and did not participate in the PD or pedagogical content assessments.

The limitations of this exploratory analysis of student item category responses and strategy use are due to the fact that these participating students make up only 22% of the students in the Jumpstart program and they do not represent all classrooms. Also, the sample of students returning parent consent forms may not have represented the population of students, but more of a subset of the population which included students who took the time to bring home the form to have their parents sign, remembering to return the form to their teacher, and whose parents were willing to take time to read or understand what they were involving their child in before signing. However, it was interesting that there were more significant changes in certain strategies, and students made more significant growth on some item categories. This information, despite its limitations, is still valuable for looking at program strengths and weaknesses to guide changes for the following year.

CHAPTER SIX: STUDENT FOCUS GROUP INTERVIEWS

Two student focus group interviews were conducted at each campus by a graduate student, each with from two to four students participating. The audio recordings of the interviews were provided to the researcher in a de-identified format at the end of the Jumpstart program for analysis. The focus group interviews lasted for approximately 20 minutes. They were semi-structured interviews that began with the same prompt, "Tell me what you have been learning about positive and negative numbers." If this prompt did not result in students talking about their understanding the following questions were used as time permitted to gather additional information about their understanding:

- What do you think about using a number line?
- How would you improve the number line activities?
- What other methods did you use to solve problems with positive and negative numbers?
- What do you think about the calculator activities? Do you prefer that method?
- If you had to rank the methods in order what would be the method you prefer the most? Least? Why?
- How would you solve the problem 5 – 12?
- If two students came up with different answers, is there anything you could do to check who is correct?

The prompt and back up questions were provided to a graduate student from The University of Texas who conducted the focus group interviews at each campus.[11] The following are sample statements shared during the focus group interviews from

[11] Originally weekly focus group interviews were going to be conducted, but the teachers felt that it would be a distraction and loss of instructional time. Several grad students had originally volunteered to assist with focus group interviews, but due to scheduling conflicts only one grad student ended up being available. So the researcher resolved to only have 2 focus group student interviews per campus to provide some background related to student's experiences with these activities related to positive and negative numbers. This is not meant to be representative of the experiences of all students in the study.

participating students to give a little background into some of the students' perceptions of the activities related to positive and negative numbers.

Using a Number Line

Students in the focus groups responded positively when asked about their use of a number line.

> Student 1: In these graphs, up and down, down is negative and up is positive so when you are talking about temperature or something or water.
>
> Student 2 : (interrupts) They make us do this to help us understand negative and positive and the direction they go in. The calculator had a program that lets us see a number line. You type in the equation and it shows you the arrows and which way it is going. It is called Number Line.
>
> Student 3: Sometimes we don't know how to show it, but if you use a number line and it says $^{-}2$ you can go to -2 and then if it says add 6 you can add 6 and see what you get. To me seeing something makes it easier.
>
> Student 4: I liked them, I like the way it works, it helps to find the difference of negatives and positives, also to find what is bigger like $^{-}5$ and $^{-}25$, we use which one is closer to zero. It can help you find the range, the difference of a number, what is in between.

From these brief statements, it is clear that for some student the number line helps them connect operations with positive and negative numbers to contexts like temperature or elevation and allows them to see what is happening. The TI-73 Numln application for the calculator was introduced for the first time this year in Jumpstart. However, only one student out of the four focus groups of students mentioned this calculator program. This program was only used in four lessons spanning four days. With additional exposure to the program and calculator vector model of integer operations perhaps more students would have referenced it in their focus group discussions. The complete transcript of the focus group interviews is provided in Appendix M.

A few students were not as pleased with the number line when dealing with problems with larger numbers or the time it took to write out the number line. The following are a few representative statements of these frustrations and how they were addressed:

> Student 5: I don't like having to write out all the negative signs. I guess you could just put a bracket with a negative above it. Because when you are taking a test you only have a certain amount of time, so you could just put a negative with a bracket above the number line.
>
> Student 6: I understand better with the pieman than the number line. The number line is too long, too many numbers.
>
> Student 7: Only use a number line for little numbers. Don't use it for extremely large numbers. If you have hard ones, I don't want to cheat and use the calculator, so maybe use pieman or try to think and figure it out for yourself, or you could make a number line and use 100, 200 that is what I did.

These statements are important to consider for curriculum and instruction decisions. The following are some questions to consider in designing a curriculum and instruction related to integers: When is use of a number line appropriate? When should students begin generalizing their understanding from the number line to more of a rule-based understanding to use more efficiently with larger numbers? Is there a way to conceptualize a number line mentally that can be used with larger numbers?

Stories and Memory Strategies

There are several references to strategies that were not in the curriculum. Some of the students referred to a memory strategy called Pieman which is a face with negative signs for eyes and a positive sign for a mouth. It is used for the rules for multiplication of integers, but some students misuse it for addition and subtraction. Others mention the use of a story or real world application such as money.

> Student 8: She (teacher) always tells us to do a story right. So you owe someone 6 dollars but you only have 5. So you subtract, I still do the subtraction so its 6 minus 5 or 5 minus 6 which is negative one.
>
> Student 9: Sometimes I get confused about multiplying positive and negative numbers. The teachers showed us about pieman. Pieman goes like something like that. A positive times a negative number, a negative and negative has to be a positive.

The mention of the use of pieman by a teacher is unfortunate, because a large part of the time was spent PD discussing the disadvantages of using pieman as a memory strategy with students, because the pilot study showed students misapplied it to addition and subtraction of operations. The use of "pieman" was mentioned by several students as their favorite strategy.

Ranking Different Strategies

When asked to rank the different strategies used for solving integer operation problems, most of the students listed the number line as their first choice. Students seemed to confuse the TI-83 daily calculator activities used for exploring algebraic patterns by entering functions, exploring tables, and exploring graphs. Several listed calculators as least favorite, because they did not like all the buttons that needed to be pressed to graph lines. Only one student referred to the calculator appropriately as the number line program that uses vectors to represent operations with integers. Again, this confusion could be because the TI-73 was only used for four days and the TI-83 was used daily for the entire program for the patterns lessons. The following are some representative examples of how students ranked the strategies they had learned:

> Student 10: I understand better with the pieman than the number line. The number line is too long, too many numbers.
>
> Student 11: The calculator is easy. But pressing all the buttons is confusing, pressing delete, gets confusing[12] In order I like the number line, then the teacher's way[13], then the calculator the least.

Rules without Understanding

One concern that continues to appear when interviewing students about integer operations is the use of rules without understanding. Student responses from focus groups seemed to attribute these rules to instruction either in Jumpstart or earlier in middle school. The following are some representative examples of this rule-based understanding:

> Student 12: $^{+}3$ x $^{-}5$ would equal +15 because a positive times a negative (pause). A Positive times a negative equals a positive and a negative times a positive is a negative. No a negative times a negative is a positive and a positive times a positive is a positive (writes out rules on paper from memory).

[12] This student earlier refers to the TI-83 patterns calculator activities where students graph linear functions. The student is referring to these calculator activities here as having too many buttons to press.

[13] The teacher's way the student refers to is described in another part of the transcript as lining up the positives on one side of a T-chart, the negatives on another side and crossing out zero pairs.

Argumentation Around Subtraction of Integers: 5 – 12

The focus group interviews ended with a sample problem for the students to solve and explain their thinking. If two or more students came up with different responses, the UT grad student was to encourage the students to justify their solution, to give them the opportunity to practice their argumentation skills. This only occurred in two of the focus groups where there was a difference between two students in the solution to this problem.

In one focus group, Student 13 argued that the answer was -7 and Student 14 believed the answer was 7. What follows is their discussion:

> Student 13: I wouldn't use a number line. I would just subtract, since you can't do it, it goes into the negative. I have to use a number line to explain it. You are at 5 and you take away 12, 1,2,3,4,5,6……12 so it would be right here, at $^{-}7$. I solve it with just regular numbers, but I explain it with the number line.
>
> Student 14: I flip it, 12 – 5 and the difference is 7. I find out which is bigger, 12 is bigger, 5 is positive, I have to take more than 5 so I have to go to the negative side. So I have to take 5 and 7 more which is 12 and I end up in the negative side which is -7.

Initially, Students 13 and 14 had different answers. When Student 14 explained his/her thinking after hearing Student 13, they ended up with the same answer. This is one common outcome of argumentation around different solutions; students share their individual strategies but come to the same conclusion resolving the difference. Student 13's response made this researcher realize a limitation of interview studies in that asking students to explain their thinking may cause them to show an understanding using a strategy they normally do not use for solving similar problems. How then, can one better understand the strategies students actually use on a regular basis?

In another focus group, Students 15 and 16 start with different solutions, but Student 16 is less confident and switches to support Student 15's answer when in reality Student 16 was correct. The following is their discussion:

Student 16: (looks at Student 15's paper) I got a different answer so now I'm confused.
Student 15: Start on the positive side and then add 12 so it's going to be 17.
Student 16: I just subtracted, and it went to $^{-}7$ rather than 17 so now I'm confused.
Student 15: I think it's 17. If you are really just subtracting then it is $^{-}7$, but if you are adding then it is 17.
Student 16. No this way over here.
Student 16. It's 17. I believe her
Student 15 That way (teacher way) is confusing to me. (uses cell phone calculator) It's $^{-}7$
Student 16. I was right, okay, I'm not confused anymore. I just looked at 5 and subtracted 12 and got $^{-}7$.

Sometime students experience either peer pressure or a lack of confidence in their own answers or in their understanding which can cause them to change their answer, even if correct, and comply with the answer of another student because students can be unsure of an answer. This is why teachers using argumentation in the classroom need to include a class debrief to have students share out their understanding so that other class members can assist with clearing up any misconceptions that develop during the small group argumentation sessions (Ryan & Williams, 2007).

These snapshots of student understanding and feelings about the Jumpstart activities related to positive and negative numbers were included to provide some insight into the students' experiences. However, these are not meant to represent any kind of overall understanding or experience of all students in the Jumpstart program. In planning for next year's Jumpstart program, this researcher recommends that number line activities need to continue to be used because of the positive response of the students and the accurate use of number lines by students to solve problems. As much as the curriculum was written to resist use of rules in the Jumpstart curriculum, teachers and students persist in using rules. Therefore, improvements in the Jumpstart curriculum and instruction are needed to help teachers and students connect rule based understanding to a more meaningful context. That way if there is any confusion when using rules, they would have a meaningful context to draw from to check their thinking. Students seemed to be comfortable using argumentation to justify their thinking when there was a difference of opinion about a solution. Continuing use of argumentation in the PD of teachers and in the activities for students is recommended to give students opportunities to clear up misconceptions and to develop reasoning and justification skills. From these few interviews, it is not clear if the vector model for integer operations or the TI-73 number line models are appropriate activities for students to strengthen their understanding since only one of the students addressed this type of strategy or model. However, it was used for only 4 of the 15 days of the program. Additional research on this model is needed to see if it is an effective model for integer operations.

CHAPTER SEVEN: CONCLUSIONS AND RECOMMENDATIONS

Research Questions and Conclusions

What follows is a review of the research questions that guided this investigation:

1. What are the general patterns of teacher content and pedagogical knowledge of integers based on responses to a pre and post assessment?
2. To what extent does PD impact teacher content and pedagogical knowledge as measured by growth between pre and post assessment?
3. Is there a statistically significant difference between the growth between students' pre and posttest scores for Jumpstart 2010 compared to the growth made by students in Jumpstart 2009?
4. Do differences in teacher content and pedagogical knowledge explain more of the variance in student performance (pretest/posttest) than years of teaching experience of the teacher?
5. What is the relationship between teacher responses' to the Constructivist Learning Environment Survey (CLES) and the student responses to the CLES?

Based on the findings from the analyses of data collected for this study, described in detail in chapter 5, the following conclusions are made:

1. The analysis of the teacher pretest and posttest PCK results showed that 77% of the teachers at pretest could not explain why multiplication of two negative numbers results in a positive number. At the posttest, this number was reduced to 48% of teachers. At pretest, 61% of teachers believed that 3 –5

was the same as 3 + ($^{-}$5). After learning about the importance of distinguishing between operations and signs of numbers, only 24% of teachers still believed these expressions were the same expressions.

2. Overall the teachers made statistically significant growth in their PCK about integers between the pre- and posttest, increasing in score from a mean of 46% to a mean of 62%. This growth is assumed to be a result of the PD; however, research by Ball and Cohen (1999) showed that teachers often make changes in attitudes and instruction after they have implemented what they have learned in their classroom. Since the posttest was given at the end of the program, the results measured what teachers learned from the start of the program to the end, which includes learning from the PD as well as learning from their experience implementing the curriculum and the argumentation strategies with the students.

3. In comparing the pre- and posttest results for students in Jumpstart 2009 to students in Jumpstart 2010, there were statistically significant greater gains made in 2010. The students started out lower than the students in 2009, but rose to about the same posttest level. In 2009, the student average gain was 6%. In 2010 the growth between pretest and posttest more than doubled to a 14% improvement. This improvement was not as great as was hoped for, but the area of integer operations continues to be a complicated instructional field because of the prior knowledge teachers and students come with that includes

misconceptions, and the short time students and teachers had during the 15-day Jumpstart program to explore these challenging concepts.

4. The HLM analysis of student posttest scores accounted for the nesting of students within classrooms, and showed that differences in teacher PCK was statistically significant and explained 27% of the classroom posttest variance controlling for years of teaching experience, student pretest score, and teacher pretest PCK score. Years of teaching experience was not significant in explaining the differences in posttest scores at the class level. The findings show that a 1 point increase in teacher PCK, the overall percent score would be associated with a .22 increase in student posttest score. The average gain in teacher PCK from pretest to posttest was 16%. For each of these percentage points, there would be an associated .22 increase in student posttest score which accumulates to a 4% increase on the class average posttest. This may not seem like much of an association, but the average growth from pretest to posttest was only 14%. So the teacher PCK was a significant variable associated with student posttest scores.
5. The results of the teacher CLES showed that teachers rated the category of Shared Control at an average of 65%, with a minimum rating of 33% and a maximum rating of 100%, which explains the extent to which students are given opportunities to explain and justify their own ideas and hear the ideas of other students. In comparing teacher responses to the responses of the participating students in their class, 47% of the teachers rated this category of

Shared Control more highly than their students, and 35% rated it the same as their students. This is one of the higher ratings that was similar between teachers and students which may mean that teachers are more realistic with what they were able to implement this summer in terms of succeeding in providing productive opportunities for students to engage in classroom discourse about mathematics.

Comparison to Findings of Previous Research

For the subsample of 70 participating students, the strategy that was the most evident from student work on the pretest was use of the number line (21%) which then became second most used compared to the more abstract symbolic rule of changing subtraction to adding the opposite of the subtrahend (second number), which was used by at least 41% of the students on the posttest based on notes on their work on the subtraction problems on the posttest. This is similar to the findings of Harvey and Cunningham (1980) in their research of grade 8 student performance and strategy use on an assessment. Subtraction of integers was the most challenging for students, and only students who converted subtraction to adding the opposite of the subtrahend (second number) were successful in solving the subtraction problems. Therefore, further research is needed to better understand student use of this strategy and whether it is more effective than other strategies in terms of accuracy for solving integer subtraction problems.

The Jumpstart curriculum starts with more conceptual representations of counters with the cancellation model and then the number line model. Then at the end of the program, students are engaged in an activity where they generalize the patterns they see into a rule. Therefore, an increase in the use of the rule on the posttest and the decrease in the use of other models (i.e., number line) may not mean that these models are not satisfactory. It may just mean that developmentally these students have moved beyond these representations and are now more comfortable with use of the rule that they generalized from their understanding of the other representations. Additional research is needed to determine whether that is the case.

According to the work of Turnuklu and Yesildere (2007), teachers need to understand the difference between a sign and an operation and the importance of assisting students in understanding the difference. In their work, pre-service teachers did not see the difference between the expressions 3 + (- 5) and 3 –5. In this study, 61% of the Jumpstart teachers on the pretest also believed that there was no difference between these two expressions. However, after the PD discussion about the purpose of operations and signs of numbers and their different roles and after implementing the curriculum with their own students, there were only 24% of teachers who continued to believe there was no difference in these expressions. The other 76% were able to explain the difference due to the operation and sign of the numbers in the problem. This may seem like a minor issue, because in actuality these expressions result in the same answer; however, students struggle with understanding the role of operations and signs, so it is important for teachers to understand the difference.

Strengths of This Study

Taking on a project such as this was a risk and a challenge, because only 6 hours of PD were available to effect teacher change, and only 15 days (1.5 hours a day) of instruction related to integers was included in the *Jumpstart* program. However, due to the small growth made in 2009 and actual loss in understanding for some students, it was important to thoughtfully plan how to research the effect of changes to the PD and the curriculum to impact student and teacher understanding to inform future *Jumpstart* program changes. A similar percent of students from 2009 completed both pretests and posttests as students in *Jumpstart 2010* which allowed the comparison of student growth each year, showing statistically significantly more growth made this year following the program changes.

All but one teacher consented to participate so that results from their pretest and posttest could be compared to determine changes in their understanding which showed statistically significant growth. All teachers completed the CLES survey reporting to what extent they believed these constructivist environment features were important and implemented in their classroom. What follows are potential limitations based on the fact that not all data were able to be collected from students because of the challenges of getting students to return parent consent forms. However, given that this is a summer program with no homework, and that students do not take backpacks to and from school, the return rate for signed parent consent forms of 102 out of 341 students was a higher rate than was expected.

Limitations

Analytic Sample of Teachers

The comparison of pretest and posttest growth for teacher PCK included only 18 of the 22 teachers in the *Jumpstart* program because one teacher did not attend the PD and three teachers arrived late and did not complete the pretest. The available data for comparison represented 82% of the teachers. Therefore, the results should be interpreted with caution because they may not represent the change in understanding of all teachers.

Analytic Sample of Students

Each year, the district faces the challenge of enrolling the students who are to be in the program by the first day of Jumpstart. Since the program is based on whether the students have passed the grade 8 TAKS test or not, many students wait until the results from the summer administration are back to determine if they will need to attend Jumpstart. Grade Placement Committees at the school are given very little time between the release of the results and the start of the Jumpstart program to meet with parents and students to explain the expectation that the students will attend Jumpstart. Therefore, approximately one third of the students in Jumpstart do not enroll until closer to the third day of the program. This is why only 216 of the 341 (63%) Jumpstart students completed the pretest.

Since the posttest covers the curriculum taught throughout the entire Jumpstart program, teachers are asked to wait until almost the last day of the program to administer it to their students, allowing at least one day for make-up testing. However, at this point in time many families take vacations which causes some students to miss the last day or two of the program, some of the students are removed from the program because they exceed the allowable number of absences or have exceeded the acceptable amount of behavior referrals, and other students decide not to attend the last few days, perhaps because of the fact that there will be post assessments. This explains in part why only 270 of the 341 (79%) Jumpstart students completed the posttest assessment. Accounting for students who entered the program late and left the program early, there were only 178 of the 341 (52%) Jumpstart students present for both the pretest and posttest assessments. This was not significantly different than the 56% of students (177 out of 316 students) who completed both pretests and posttests for Jumpstart 2009. However, the findings of this study are based on the outcomes for only a little more than 50% of the students each year; therefore, the results cannot be assumed to be true for all students in the program, including students who enrolled later and students who left the program early.

Specific Sample Characteristics

The results of this study apply to the specific students enrolled in this Jumsptart program which are students in eighth grade transitioning to high school who have not passed the TAKS mathematics test and are located in an urban school district in Central Texas. The goal of this study was not to draw conclusions about or make generalizations to any larger population in the State of Texas or the nation or for any other students with different characteristics. The goal was to compare the results to similar students in the previous Jumpstart 2009 program to understand if the growth made during the program was significant and to inform the development of the next Jumpstart 2011 program.

Changes in Teacher Knowledge

Even though the findings showed that the PD had an effect on teacher PCK, the results are based on an instrument designed by this researcher based on prior research, but may have limitations due to validity and reliability issues. No test of validity or reliability was conducted on this measure due to the small sample size of participants in the pilot study development of the instrument (n = 13) and in the actual sample taking the assessment as a pretest and posttest (n = 18). The assessment was closely aligned with the PD and the curriculum the teachers were to implement. So interpretation of the findings should not assume that the outcome measure represents a complete assessment of a teacher's PCK in mathematics or even specific to integers in a broader mathematical sense.

Changes in Student Understanding of Integers

The student outcome measure used was the pretest and posttest given each year for the Jumpstart program that was part of the *Navigator: Positive and Negative Numbers* (America's Choice, 2009) curriculum that teachers were asked to give all students for accountability purposes to measure progress for students participating in the program. This measure does not have any reliability or validity information from America's Choice since it is a newly developed product. Some of the other Navigator curriculum modules, such as the one with rational numbers, have been around longer, have been used in other research studies, and have statistically significant evidence of their impact on student performance[14]. However, the goal was to use this data as an artifact of the program, provided in a de-identified format from the district to understand changes in student understanding of integers between the pretest and posttest. If there was another outcome measure chosen, only the data for the 102 consenting students would have been analyzed. So the use of the *Navigator* assessment data allowed for a more representative result of the effect of student participation in the Jumpstart program on student performance.

Constructivist Learning Environment Survey

Since the CLES was not part of the Jumpstart program, it was an optional survey that only students with parent consent and student assent completed. There were only 102 students with consent and only 71 who completed the CLES survey with a few students leaving items unanswered, so that only 63 students had complete ratings for all items. These participating students who took the CLES represent only 18% of the *Jumpstart* 2010 students. Therefore, the summary of student responses to the CLES should be interpreted with caution, because they may not represent the experiences of all of the Jumpstart students. Also, there were several classes which did not have any consenting students that completed a CLES survey, so the comparisons of similarity between teacher and student responses should also be interpreted carefully, understanding that the pattern of similarity between teachers and students for certain items only represent that pattern for the small group of students that participated and for their respective teachers.

[14] For more information, visit the America's Choice website at http://www.americaschoice.org/resultsmathematics.

Implications and Future Directions

Implications for Professional Development

The PD design of this study was informed by research such as the one conducted at Riverside Middle School by Mundry and Loucks-Horsley (1999) which documented that implementation of a reform mathematics curriculum that required PD was a balance between the practical issues that concern teachers such as being prepared to implement curriculum activities and the new instructional practices and beliefs that were part of the reform curriculum. This researcher believes that the changes made in the PD from 2009 to include less of a focus on the curriculum activities and more of a focus on teacher conceptual understanding of integer operations through use of argumentation had a positive impact on teachers and students based on evidence that there were instructional practices occurring which allowed students to engage in conversations about mathematics based on teacher and students CLES responses. However, the PD only lasted six hours.

Haycock and Robinson (2001) conducted research which showed "PD that makes a difference for minority students is PD that deepens teachers' knowledge of the curriculum they are teaching, helps them find or create effective lessons, and enables them to assess and respond to student performance" (p. 18). Impacting a teacher's ability to assess and advance student learning was an area that could have used more emphasis and is something to consider in the development of the Jumpstart 2011 program.

Implications for Pre-service Teacher Preparation

In analyzing the pretest item performance of the Jumpstart teachers, many of the teachers came to the program unable to explain why the answer to the problem $^{-}5$ x ($^{-}8$) is positive (72%), how to solve the problem 5 – ($^{-}8$) beyond just changing it to an addition problem (50%), and the difference between the expressions 3 –5 and 3 + (-5) which was true for 61% of the teachers. Instructors of courses for pre-service teachers should consider these entry level points as potentially areas that their pre-service students may share and find ways to explore integer operations to improve understanding of these operations conceptually as well as pedagogically. The Jumpstart PD caused an improvement across teachers in their understanding of these problems, but 48% of the teachers were still unable to explain why $^{-}5$ x ($^{-}8$) is positive and 43% were unable to describe how they would explain the solution of 5 – ($^{-}8$) to a student. Additional research is needed to continue to improve teacher understanding so that teachers will be more confident in addressing similar questions from students in their class.

Implications for Equity

The concern that led to this dissertation study was that the students in *Jumpstart 2009* who were in classes taught by first-year teachers made less growth and some even lost ground between pretest and posttest on the measure of understanding of integers. Research has confirmed the importance of having highly qualified teachers (defined as the proportion holding state certification and the equivalent of a math major), especially in schools that serve low-income students with a previous history of low achievement. Research by Darling-Hammond (1999) using National Assessment of Educational

Progress (NAEP) student achievement data and state data on teacher major and certification showed that the less advantaged students were less likely to have teachers that are fully certified and hold a degree in their field and more likely to be taught by a teacher who started teaching uncertified. The results of Darling-Hammond's study also suggest that teacher quality is related to student achievement outcomes even after controlling for student poverty and language background. However, in some situations, such as this short-term summer program, it is difficult to recruit highly qualified teachers. Therefore, the results of this research show the potential value added of including PD focused on improving teacher PCK to improve performance for students who are not afforded the opportunity to work with highly qualified teachers. The results of this study showed that teacher PCK was statistically significantly associated with student achievement even when controlling for years of teaching experience.

However, what actually occurred in the classroom in terms of opportunities to participate in talking and writing about reasoning about mathematics is not clear. The results of the teacher CLES showed that teachers rated the category of Shared Control at an average of 65%, which explains that students are given opportunities to explain and justify their own ideas and hear the ideas of other students; however, individual teacher ratings ranged between 33% and 100%, showing that some teachers were not as successful as others in facilitating classroom discourse about mathematics. Based on the findings of the 1993 National Survey of Science and Mathematics Education (Weiss, 1997) low-ability students were provided with fewer opportunities to engage in inquiry activities or to write about their reasoning while solving math problems. The goal was for the PD and curriculum unit for this study to improve opportunities for the low-performing grade 8 students enrolled in *Jumpstart 2010*. However, it appears that not all students were provided with these kinds of opportunities. Therefore, it remains to be determined what kind of support is needed for teachers to provide these kinds of opportunities for their students.

Future Directions

There are other exploratory analyses that could be conducted with this data set, which are outside of the research questions for this study, but that might prove interesting to the field, such as, whether or not there was differential growth between pretest and posttest for students in different percentiles comparing 2009 students to 2010 students and also comparing students with teachers with high or low PCK. This would examine whether the changes to the program and/or the changes in teacher knowledge affected some groups of students more than others.

Additional research is needed on measures of Teacher PCK that are more content specific. For example, the Mathematical Teacher Knowledge measure (Ball & Hill, 2009) could have been used to assess teacher knowledge, but it would cover a variety of mathematical topics. For this study, the researcher was looking at integer understanding specifically. Additional research and development could be performed of evaluation measures that are topic specific and include items that measure teacher knowledge about the topic content in mathematics, knowledge about students (i.e., common misconceptions), and knowledge about instruction related to the selected topic. This kind of evaluation measure would be beneficial for determining outcomes of short PD experiences related to curriculum implementation for a specific unit of study.

Additional research is needed to explore the kinds of support teachers need in order to facilitate classroom discussions about mathematics, small group discourse and argumentation, and opportunities for students to participate in inquiry activities. The PD and curriculum for Jumpstart were not sufficient to provide all students with these kinds of opportunities. There were classrooms where this was happening based on a comparison of student and teacher responses to the CLES, and there were classrooms where less of this was happening. Beyond PD, there must be support such as mentoring and model teaching that could be used to advance teachers' practice in this area.

Between pretest and posttest there was a change in strategy use on the test as evident in the work of the 70 participating students. The change in strategy use was a move away from the numberline and cancellation with objects model towards more of an abstract-rule based strategy of changing subtraction to adding the opposite of the subtrahend. Further research is needed to determine if instructional practice influenced this strategy use or if it is more developmental as students generalize from patterns they see with the other representations, they move towards using an abstract symbolic rule for solving problems with subtraction of integers.

General Discussion

The topic of integers and integer operations was chosen because of its importance as a foundational number and operations concept, the research evidence of persistent student misconceptions (Ryan & Williams, 2007), and the challenges faced by the mathematics community to agree on an effective model to support students' conceptual understanding that is also comprehensive and comprehensible. Through the process of gathering information on potential instructional strategies and the research behind their effectiveness, research on teacher PCK seemed woven throughout as authors discussed the importance of ensuring that teachers have knowledge of mathematics, knowledge of representations, knowledge of students, knowledge of learning and cognition, and knowledge of teaching (An, Kulm, & Wu, 2004; Fennema & Franke, 1992; Shulman, 1995; Turnuklu & Yesildere, 2007). Therefore a decision was made to focus less on which instructional model is best for teaching students about integers and integer operations and to focus more on researching teacher PD to improve teacher PCK related to integers.

Originally, the focus of the PD was to ensure that teachers were aware of common student misconceptions, but after further review of the literature, it was an almost impossible task to review all of the common misconceptions. It was at that point when a review of literature on argumentation and student discourse about mathematics informed the final focus of this study as a less teacher-directed intervention to challenge misconceptions and a more student-centered intervention to encourage students to share their thinking and their strategies and for students to work through helping each other come to a stronger understanding of integers. However, first teachers needed to experience argumentation by expressing their understanding of different strategies for representing integer operations and then coming to understand a new representation with the vector model to deepen their understanding. This seemed to be an appropriate intervention, since these students just finished grade 8, so integers are not a new concept to them. They have prior knowledge and experience with integers to bring to the discussion. The challenge remained to confront teacher beliefs about a student's ability to engage in productive conversations about mathematics by providing PD that modeled classroom structures and norms for productive argumentation.

The observations made during the PD were that most of the teachers were engaged in the activities where they experienced argumentation around representations of addition and subtraction of integers. There was evidence during the stations activities, that teachers were already making connections as to how they could use certain activities and certain questions to form the problem that would be the center of the argumentation for the students. However, there was also evidence of teachers with rule-based beliefs that the only way students learn is by memorizing and practicing rules until they are automatic. On the teacher PCK posttest, several teachers shared experiences with students resisting participation in discussions due to low confidence in mathematics and a concern about being wrong. Others shared the challenges of facilitating small group discussions when one of the students was overly confident and verbal, causing the other students just to listen and not share their ideas for fear of being wrong. Twenty-four percent of the teachers admitted to not engaging students in argumentation. If this research project had lasted all year, teachers would have had more time to build the classroom culture and expectations for productive conversations and more time to get to know the students to find ways to group different students to encourage more participation.

One concern is that close to 50% of teachers on the posttest continued to believe that the only way to understand why $^{-}5$ x $^{-}8$ was positive was because of a rule that when one multiplies two negatives (or an even number of negatives) one gets a positive, despite their exposure to a number line model of multiplication as repeated addition and subtraction and the representation of multiplication with colored counters during the PD. Only two of the 21 teachers that completed the demographic survey stated that they had a major in math, so perhaps their exposure to integers may be limited to the way they were taught in mathematics in school, possibly with rules. Since that method has worked for them, there was some pushback about why these other representations should even be considered. Just six hours of PD and three weeks of a required day-by-day curriculum developed to expose students to reasoning about mathematics and use of multiple representations may not have been sufficient to change some of the teachers' beliefs about a student's ability to engage in productive conversations about math and beliefs about teaching integers.

Ball and Hill (2009) conducted research in classrooms to observe and identify "common tasks of teaching that require mathematical skill and insight" (p. 69). They found that mathematical understanding for teachers involved the following: "posing questions, interpreting students' answers, providing explanations, and using representations" (p. 69). Ball, Thames, and Phelps (2008) investigated Mathematical Knowledge for Teaching (MKT) and described what they discovered in the model that had two forms of knowledge: subject matter and PCK. On the subject matter side, the knowledge was divided among three domains: common content knowledge, knowledge at the mathematical horizon, and specialized content knowledge. On the pedagogical content side, the knowledge was separated into these three domains: knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum. At the time of the development of this dissertation study, this literature had not been reviewed. So the term used throughout this paper to refer to the kind of knowledge that is important for teachers to possess was PCK. However Ball and Hill's (2009) model of MKT is one to further consider because it seems to get at some of the teacher behaviors that support argumentation in the classroom.

Ball and Hill (2009) expressed that this MKT model gets at some of the important aspects of teacher knowledge that are in Shulman's (1986, 1987) description of PCK. After completing this conceptualization, as part of the Learning Mathematics for Teaching Project, Ball and Hill created assessment items called Mathematical Knowledge for Teaching (MKT) measures and administrated the questions to large groups of teachers. The questions have also been used by other researchers and PD projects. Through this research, Ball and Hill found that the MKT of teachers was strongly associated with the quality of their instruction, specifically the following: "use of mathematical explanation and representations, responsiveness to students' mathematical ideas, and ability to avoid mathematical imprecision and error" (p.70). Earlier research by Hill and Ball (2004) had found that teachers who participated in a summer PD that focused on teachers' use of mathematical representations, explanation, and communication performed better on MKT measure than teachers at similar sites who did not receive this kind of focus.

The sample items for the MKT measure[15] were reviewed for this study to determine if any would be appropriate to include on the teacher PCK assessments, but most of the sample items related to operations with fractions and whole numbers. The two items that included integers did not seem appropriate because they were related to concepts of solving inequalities and the associative and distributive properties which were not addressed in the *Jumpstart Program.* To have access to additional items, The Learning Mathematics for Teaching Project recommends travel to a training for researchers that occurs every few months for free. Therefore, it was decided that this measure would not be used for this study, but is something to consider for future research of teacher knowledge.

In conclusion, the professional development and curriculum modifications made this year for Jumpstart 2010 resulted in statistically significantly positive gains in teacher PCK and student achievement. The change in the professional development, compared to the previous year resulted in less of a focus on student activities and more on developing teacher PCK. As a result, teachers who were originally unable to explain why multiplying two negative numbers results in a positive number, were now able to explain conceptually why this is true when they completed the posttest. Other teachers who did not differentiate between an operation of subtraction and the negative sign of a number at pretesting made improvements in understanding the distinct roles of each of these components of a mathematical expression. The resulting positive change in teacher PCK

[15] Sample items are available through The Learning Mathematics for Teaching Project at http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf

was associated with positive gains in student understanding on the posttest. To have such a positive effect on teachers and students in just six hours of PD and three weeks of a curriculum and instruction intervention is encouraging for future PD development.

Appendix A: Integer Unit Daily Overview

Day	Description	Materials
Day 1	Lesson 1: Pre-test Work Time Discussion T-Chart: Correct/Incorrect	Copies of Student Pre-test Student Journals Poster Paper T-Chart
Day 2	Lesson 2: On a Number Line *Discussion*: #'s 1C, 1D, 5C, 5D Confronting Misconceptions or Competing Strategies depending on student work Thermometer Problem *Discussion:* What is Math Reasoning & Argumentation? Establish Group Norms	Sample Student Misconceptions (Lesson 2) Thermometer Points of View Argumentation Sentence Stems Poster Group Norms Poster
Day 3	Lesson 3: Understanding Positive & Negative Numbers Vocabulary: Greater, Less, Opposite, Ascending Order, Descending Order Lesson 4: Greater Than Less Than Game *Discussion*: Which is greater -1/5 or -0.23 Add to group norms, add to sentence stem poster	Sentence Stem Poster Group norms Poster Lesson 4 Group Cards Problem for class discussion Record Sheet for Discussion
Day 4	Lesson 6: Working with Positive and Negative Numbers *Discussion*: Misconceptions from Checkpoint, students share reasoning/argumentation Temperature Changes Activity Lesson 6 *Discussion*: strategies for determining difference in temperature, meaning of difference.	Copies of Checkpoint Student Reflection Sheet Sentence Stem Poster Group norms Poster Lesson 6 Group Cards Misconceptions from Checkpoint (Anonymous)
Day 5	4 Corners Student Activity to assess prior knowledge and strategy use. *Discussion*: Argumentation using Competing Strategies Lesson 7: Adding Positive and Negative Numbers *Discussion:* What are these number lines	4 Poster Paper with identical problem (-8) + 5 Markers Lesson 7 Group Cards

Day	Description	Materials
	showing? How else can you add integer on a number line? How else can you add integers? What are the advantages/benefits of using one strategy over another?	
Day 6	TI-73 NUMLIN Activity 1: "Integers It All Adds Up" *Discussion:* Why is it that 13 + 8 is shown as 0 + 13 + 8? How do the "number rays" or "vectors" represent the operation of addition? What does the length and direction of the vector mean? What happens when you add same sign numbers? Different sign numbers? Does order matter (commutative)? Students predict, sketch, and check with calculator	TI-73 calculators Activity 1 handouts Calculators
Day 7	Vocabulary: addition, plus, minus, subtract, positive, negative, take away, difference, distance, length, operation vs. sign *Discussion*: How do you read these problems? What is an operation? What is a sign? Lesson 8: Subtraction on the Number Line" *Discussion:* (-8) - 11 and 8 - (-11) what do they mean? How do they look on a number line? Does order matter (commutative)? Is there another way to solve the problem besides a number line? Which strategy do you prefer? Why?	Vocabulary Posters (+, -) Lesson 8 Group Cards
Day 8	Lesson 9: Adding and Subtracting Game *Discussion:* Did you get any expressions with solutions that had a difference of 0? Any that came close to 0? What strategy did you use to find the difference?	Lesson 9 Group Cards Student Self-Reflection

Day	**Description**	**Materials**
Day 9	Checkpoint 10 Misconceptions: *Discussion:* Students discuss if they agree or disagree and why. Take time to discuss misconceptions from Checkpoint 10 completed in Center 1 on Day 8. Have students practice argumentation and justification related to the sample misconceptions you provide the class.	Sample misconceptions from Checkpoint 10 (anonymous)
Day 10	TI-73 NUMLIN Activity 2 *Discussion:* #6 Predict the sum of 39 and -45. How is it similar to 45 - 39? #10, 18 - (-13), How can you start with 18, subtract -13 and then end up with a bigger number (31)? Isn't subtraction take-away? Shouldn't the answer be smaller? #11-13 what do they notice about subtraction on the calculator? Journal Activity: For subtracting integers which model do you prefer and why? (Difference model, vector movement changing subtraction to add the opposite, or other model) *Discussion of Journal Activity Responses*	Copies of NUMLIN Activity 2 Calculators Journal Activity
Day 11	TI-73 NUMLIN Activity 3: Multiplying Integers *Discussion:* Which do you prefer, the Integer Chips/Pieces Modeling of Multiplication of Integers or the calculator vector model of integers? Why? #10 discussion of student responses Lesson 12: Multiplying and Dividing *Discussion:* What pattern did you notice? What rule did you create to describe the pattern? What are the benefits of having rules? Journal Activity: For multiplying and dividing integers which model do students prefer and why? (chips model,	Copies of TI-73 NUMLIN Activity 3 Calculators Journal Activity

Day	Description	Materials
	vector model, rule) *Discussion of Journal Activity Responses*	
Day 12	Post-test Journal Activity: Mathematics as finding patterns, developing rules to describe those patterns, and applying those rules to other examples. What have you learned over the last two weeks about integers? Are there any rules you have learned that you can apply to new problems? *Discussion of Journal Activity Responses*	Copies of Post-test Journal Activity
Day 13	Time reserved for teachers to catch up if they got behind and need additional time to complete lessons or center activities.	
Day 14	Time reserved for Dimension M tournament and Jumpstart certificate distribution.	

Appendix B: Sample Misconceptions Prompts for Discussion

The following sample misconceptions were provided as part of the Jumpstart curriculum to be used as prompts for argumentation in the classroom if the available student work samples did not provide enough differences to base a discussion. I wrote these prompts so I provide them here as a resource for work with students around misconceptions using argumentation. These are based on common misconceptions from the pilot study I conducted in May 2008.

Comparing Numbers

Question: How do you know if one number is larger than another number when you are asked to compare 2 integers?

Student: "I always remember that if it's a negative and a positive, the negative will always be the larger one. So if it's a negative 1 and a negative 9, the negative 9 would be larger than the negative 1 because the negative 9 is the biggest."

Adding Integers

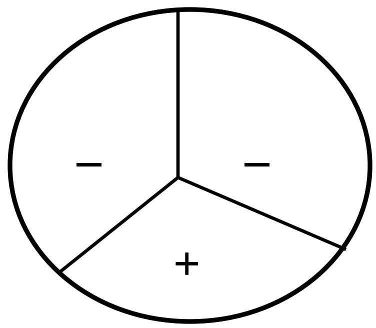
Question: (-5) + 8 = ?

Student: 5 + 8 is 13, so it would be negative, no just positive 13. I think its positive because that's the biggest number.

Question: (-3) + (-6) = ?

Student: I was thinking about the negative sign. I could be wrong. I think it's positive 9, because a negative times a negative makes a, NO, a negative times a negative makes a negative, and a negative and a positive makes a positive number. I just know that it's 9, Positive. Because two negatives equal a positive.

Student Drew Pie Man shown below:



Subtracting Integers

Question: 2 - 7 = ?

Student: When I was in kindergarten, I didn't think you could do this problem. Now I know this is the same thing as 7 - 2, so the answer is 5.

Question: (-3) - 5 = ?

Student: It is -2, because you just subtract.

Another Student: You can draw 3 boxes or 3 circles and you can tell you are trying to get to 5. You are missing 2. So the answer is 2.

Multiplication of Integers

Question: (- 4) x 5 = ?

Student: It is 20, a negative. No it's a positive, because the negative is in front of the 4, which is smaller and the 5 is positive and it is larger. So it is positive 20.

Another Student: I just multiply that times that and get 20, and since the higher number is positive, it's going to be positive.

Appendix C: July 8, 2010 Professional Development Overview

8:30am - 3:30pm

8:00 - 8:30 Registration & Breakfast

8:30 - 8:45 Teacher consent form & pre-assessment

8:45 - 9:00 Share Participant Prior Knowledge

What makes integer operations challenging?

What are some common misconceptions?

What strategies have they used to teach about integers?

What experience do they have with students using argumentation or justifying thinking?

What concerns do they have about facilitating class discussions?

Vocabulary for integer operations?

Posted on the Wall: Goals for Today briefly point out to the class

Goals for Today:

Why Argumentation? Results from Jumpstart 2009 (little to no progress)

NCTM new emphasis on Mathematical Reasoning

Experience argumentation through lessons from the Integers Unit

Argumentation as Competing Strategies & Confronting Misconceptions

Scaffolds to support Argumentation in the Classroom:

Conversation Stems

Groupwork Norms

Organizing Productive Group Argument in Discussion

Norms for Class Discussions

Keeping a Record of the Discussion

Student Self Reflection

Reflecting on the Experience (Q & A)

9:00 - 9:15 Part 1: Introduction:

Overview Jumpstart 2010- Curriculum, Lessons, Centers

Overview of data collected from Jumpstart 2009

Areas in need of improvement from Jumpstart 2009

9:15 - 10:15 Part 2: Why Argumentation?

Jigsaw Parts of Ryan & Williams (2007) "Children's mathematical discussions."

Everyone reads selection 1 and then jigsaw selections 2-5.

Selection 1: Argumentation in discussion: persuasion through reasoning (pp.31-33) (What is argumentation in math classrooms?)

Selection 2: An analysis of mathematical argumentation (pp.34-39) (What are argumentation moves? Give examples)

Selection 3: What makes a good mathematical argument (pp.39-40) (What are the key components of a mathematical argument?)

Selection 4: Maintaining dialogue in practice: social interaction (pp.41-44) (What are the types of teacher moves to facilitate argumentation in class discussion)

Selection 5: Small Group Dialogues in the classroom (pp.44-49) (How would you respond to a teacher who is concerned that sharing misconceptions might make misconceptions contagious?)

Groups create poster to address their question and share a brief summary of the section they were assigned to read. Groups share out their thinking.

10:15 - 11:15 Part 3: 4 Corners Activity (Adding Integers)

Group work Norms-Brainstorm List among participants (Poster)

Complete 4 corners activity in groups

Share and discuss the different strategies

Misconceptions: What are the common student misconceptions (create poster) Discuss Norms for Argumentation (How are we going to have this discussion?)- Social Norms (i.e. respect) and Mathematical norms (what makes a good mathematical argument, how do you evaluate/critique a mathematical argument?) (create poster) [NCTM Middle School Math Journal has examples of arguments that are of different levels of quality and justification]

Show 1 example of America's Choice number line model for addition and hold an argumentation discussion. Question: Why did America's Choice come up with this model? Focus of Part 1 Math: does it work for solving problems (reference criteria for good mathematical argument). Focus of Part 2 Pedagogy: Does it work for teaching students about integer addition?

Reflect on the argumentation experience (record conversation stems used during the discussion). Question 1: What is the value of argumentation for learning about math? Question 2: How can you support argumentation in the classroom? (What structures were modeled today as examples of support structures?) Discuss Example of Keeping a Record of the Discussion using the

Adding Integers (Note: Record can show where the argument left off, because it may not be resolved at the end of the day)

Misconceptions about adding integers Handout: Question: What is the student's misconception? What would you say to the kid? Argumentation often brings out student misconceptions, what do we do? Participants practice with a partner to assess and advance student thinking based on the sample student misconception.

11:15 - 12:15 Part 4: Adding & Subtracting Integers on a Number Line

Given a subtraction problem, how would you represent on a number line? Give half problem a and half problem b without telling them the difference [a. (-8) – 11 and b. 8 – (-11)] Pairs form groups of 4 (one pair with problem a and one pair with problem b). Groups of 4 Discuss and converge on one strategy. (Note: Researcher walks around modeling ways that teachers can facilitate small group argumentation)

Meta-conversation about strategies for supporting this sort of activity in small groups (reference the article read about small group argumentation)

Discuss why they drew the number lines the way they did (Some may be using the same kind they did in the 4 corners activity, others may be influenced by the America's Choice model for integer addition)

Researcher talks about skipping the whole class synthesis argument for time, but in their classes they should take time to have small groups share out.

Show America's choice model for subtraction if no one else came up with it. Hold Argumentation session about this model. Focus of Part 1 Math: does it work for solving problems (reference criteria for good mathematical argument). Focus of Part 2 Pedagogy: Does it work for teaching students about integer subtraction?

Misconceptions about subtracting integers Handout: Question: What is the student's misconception? What would you say to the kid? Why isn't subtraction commutative? Participants practice with a partner to assess and advance student thinking based on the sample student misconception

12:15 - 12:45 Lunch

12:45-1:45 Part 5: Calculator Explorations with TI-73 (+ , - , x)

Teachers explore TI-73 Integer Addition and Subtraction Activity.

Discussion: Which strategy do you prefer- Navigator number line, TI-73 number line, or any of the others shown today? Why? Part 1: Mathematical, Part 2: Pedagogical Question: How would you show Multiplication of integers with chips? with a number line? Calculator Exploration of Integer multiplication with TI-73. Which strategy do you prefer- chips or the number line for multiplication? Why? Part 1: Mathematical, Part 2: Pedagogical

1:45-2:00 Part 6: Vocabulary

Discuss meaning of the different words used in integer operations: addition, plus, minus, subtract, positive, negative, take away, difference, distance, length, operation vs. sign. Discuss vocabulary, how do you read these problems?

Especially what is an operation? What is a sign? Discuss misconceptions in how students might read these problems. Is subtract a negative really the same as add a positive? Why? Is subtract a positive really the same as add a negative? Why

2:00 – 3:00 Part 7: Integers Centers

Teachers rotate through 6 centers spending about 8 minutes at each center to engage in the activity the students will be using for the integer center. Model center facilitation as a teacher by asking questions, and encouraging students to talk and engage in argumentation related to the center activity.

3:00 – 3:15 Part 8: Reflecting on the experience, Questions & Answers

(Add new learning to posters from the morning)

What makes integer operations challenging?

What are some common misconceptions?

What strategies have they used to teach about integers?

What experience do they have with students using argumentation or justifying thinking?

What concerns do they have about facilitating class discussions?

Vocabulary for integer operations?

3:15 - 3:30 Part 8: Overview of Jumpstart Integer Lessons & Center Activities:

Appendix D. Teacher Content & Pedagogy Assessment

1. List as many real world examples for integer operations that you can think of.

2. Provide as many examples as you can of how integers are used in other domains (i.e., physics, chemistry, animation, forensics, etc.). Be as specific as possible of your understanding of the use of integers and integer operations within that domain.

3. Given 5 - (-8) = ? How would you explain to a student the answer you got?

4. Given -5 x (-8) = ? Why does the answer have the sign that it does?

5. A student teacher reads (-6) + (+7) as "minus six and plus seven" and 6 - (+7) as "six minus plus seven". Is this appropriate? Explain why or why not.

6. A student when given the problem 4 - 7 = ? responds 3. What is a possible misconception that they might have that causes them to have this response? How would you respond to them?

7. Is 3 - 5 the same as 3 + (-5)? Why or why not? Explain.

8. Have you ever supported your students in arguing about math? Describe the experience, any benefits, and any challenges below:

Appendix E: Teacher Demographic Survey

Dear Teacher,

Thank you for participating in the study of the effectiveness of 2 replacement units for teaching students about positive and negative numbers. I would appreciate your taking time to provide a little demographic information for research purposes. Your response to this survey will only be used for general statistical purposes to better understand the group of teachers in the study. The final report will summarize the results across all the teachers participating and will not provide any information that would identify you to your school, administrator, or district.

1. Name: ____

2. Age: ______________

3. *Jumpstart* Campus: __________________________

4. Gender (check one) ☐ Male ☐ Female

5. Total years teaching experience: __________

6. Have you taught in the *Jumpstart* Program Before? (check one) ☐ Yes ☐ No

7. Highest degree (only check one)
   - ☐ Bachelors Major:___________________________
   - ☐ Masters Major:____________________________
   - ☐ Ph.D Major:______________________________

For Study Purposes only: Teacher Study ID: _________________________

Appendix F: Student Assessments

The Pre- and Post Assessments were part of the America's Choice (2009) *Navigator: Positive and Negative Numbers* program purchased by the school district. These materials are copy written so they are not provided in this report. What follows are some similar problems so that the reader can better understand the types of problems students encountered on these assessments.

Mid-Assessment

Mark and Label the following numbers in their correct position on the number line below:

(1) -5 (2) - 0.5 (3) 2 (4) $\frac{1}{2}$ (5) -2.5

←---→

Choose the correct symbol to write between each pair of numbers

< "is less than" or > "is greater than"

(6) 2 ________ -1.5

(7) 3 ________ - 2

(8) 4 ________ -5

(9) -2 ________ -1

(10) -2.5 ______ -3

Solve the following addition problems:

(11) (-6) + 5 =

(12) (-3) + (-8) =

(13) 7 + (-5) =

(14) 6 + (-12) =

(15) (-9) + 4 =

Solve the following subtraction problems:

(16) 3 - 8 =

(17) (-6) - 3 =

(18) 5 - (-2) =

(19) (-4) - (-5) =

(20) (-10) - 4 =

Describe if the following are true or false:

(21) (-5) x (-4) = 5 x 4

(22) 7 x (-3) = -7 x 3

(23) (-6) x 5 = -6 x -5

(24) 6 x 4 = -6 x -4

(25) (-2) x (-2) = 2 x 2

Appendix G: Rubric for Scoring Teacher PCK Assessment

Question	Examples of Low Rating (0)	Examples of Medium Rating (1)	Examples of High Rating (2)
5 – (-8) explain to a student the answer you get PreTest #1 Posttest#3	Explains with a rule or any statement that does not make a connection for this procedure but just uses a rule such as adding a negative number will consistently result in the same answer.	Chip model or metaphor (debt story) but only one example not clearly explained.	Number line with inclusion of direction and magnitude type of model possibly in addition to another model or rule helping the student make a connection.
-5 x (-8)? Why the sign PreTest #2 Posttest#4	Does not know Rule "negative times negative is a positive"	Some understanding of an example, uses the word "opposite" but does not include the concept of repeated addition. So explains the negative but not the operation.	Higher level understanding of why, either as repeated addition or the use of the word "opposite" For example -5 x -8 is the opposite of 5 x -8 and 5 x -8 is repeated addition of -8 five times. Explains the negative and the operation of multiplication
(-6)+ (+7) and 6 – (+7) student teacher PreTest #3 Posttest#5	There is no difference, or not much of a problem, or some other error in thinking, i.e.	Interchangeable, but one is more precise. Getting at the idea of an operation, but not clearly stating it .	Role of subtraction as an operation vs. signs, the two result in the same answer but they are two different types of problems, they have two different operations, which means something different.
4 – 7 = 3 PreTest #4	No mention of misconception that	Commutative property with either no teaching	Addresses misconception that

Question	**Examples of Low Rating (0)**	**Examples of Medium Rating (1)**	**Examples of High Rating (2)**
Posttest #6	subtraction is commutative property and no teaching strategy.	approach or unsure of teaching approach. ie. Or teaching strategy but no mention of mention of misconception that subtraction is commutative.	subtraction is Commutative and includes a teaching approach.
Is 3 – 5 the same as 3 + (-5)? PreTest #5 Posttest#7	Not sure or yes they are the same but not sure why.	Explain why equivalent but don't mention the change in operation. May include a description of +(-#) as multiplication and that when simplified they becomes equivalent.	Understands the importance of operations and any math concept that makes them different, such as inverse operations.
Argumentation PreTest #6 Posttest#9	No experience with students using argumentation in the classroom.	Minimal, or mixed experience, not quite at the level expected. May just refer to as class discussion.	Clear past experience and outcome that sounds like the expected involvement of students in argumentation
Real World Examples PreTest #7 Posttest#1&2	No real world, no domain example, no explicit reference that relates to negative numbers	Either real world or domain example(s) (Besides math domain)	Both real world and non-math domain examples
Number line Posttest #8 (Not included in Assessment score but included for exploratory purposes)	Don't connect number line to operations, i.e.,	Mention either position, magnitude or direction, but not all in connecting with integer operations.	Mentions at least two of position, magnitude, direction and connects to integer operations.

Appendix H: Student Focus Group Questions

Number Line Replacement Unit

This focus group discussion will begin with the following statement to begin the discussion, "Tell me a little about what you have been doing in your class with positive and negative numbers?" Then the following questions will be used as needed if these areas are not brought up in the discussion.

1. What did you like the most about using a number line to solve integer problems?

2. What did you like the least about using a number line to solve integer problems?

3. How did you like the calculator activities where you graphed integers as vectors?

4. If you were given an integer problem, such as 5 - 12, how would you go about solving the problem?

5. Do you have any suggestions for how to improve these number line activities?

Appendix I: Real World and Domain Applications of Integers

The following represents the diverse background and experiences with integers represented by the Jumpstart 2010 teachers taken from the pre and post PCK assessments. This list is a great resource for a variety of problem contexts for curriculum development in the area of operations with integers.

Some Real World Examples:
Stock market
Temperature change
Rate change [leading later to velocity and acceleration]
Whole dollar bank deposits and overdrafts
Football field
Elevation
Sea level
Golf scores
Taking a trip (algebra project)
Elevator
Giving change,
Doing taxes in TurboTax
Sports scores
Measuring weight loss and weight gain
Inflation of tires is almost always a whole number (PSI) that you're trying to get close to
Gambling
Card games such as Euchre

Chemistry
Counting number of protons/neutrons/electrons
Ionic charge, e.g. +3, -2
Number of atoms and molecules in a chemical reaction formula, e.g. $2H_2 + O_2 \Rightarrow 2H_2O$
Working with temperature in analyzing any Chemical reaction involves an understanding of integer number lines.
Conversions of units involves multiplication and division of integers

Media and Film
In filmmaking, frame rates (i.e. 24 frames per second) involve an understanding of how fast or slow the resulting film will be. So, increasing from 24 will use more film and therefore make the action seem slower, and decreasing from 24 will use less film and therefore make the action seem faster. For instance, shooting 48 frames per second will make the result look like it's at half speed. And shooting at 12 frames per second will make the result look it it's playing at double-speed.

Art and Design
When you edit something in *Photoshop*, there are several settings in which the default is zero, and you can move a cursor back and forth to a positive or negative number to change certain properties of the picture, such as the contrast, saturation, hue, brightness, etc. Also, when you rotate objects, you are asked to choose the angle to rotate them, and you can choose positive or negative angles.

Animation
Translating objects from one coordinate to another, so you're adding or subtracting integers to the x,y,z coordinates. And if you want to "grow" something, then you would multiply appropriately by an integer

Forensics
Blood volume in forensics

Physics
Acceleration
Velocity
Force vectors
Conversions of units involves multiplication and division of integers

Finance
Debits/credits on accounting sheets

Economics
The slope of a demand curve

Applied Math/Statistics
When determining relationships between variables, parameters or constants might be negative
Probability – an expected value can be negative in a game where you lose money

Computer Science
Loop counter to figure out how many times you have repeated a loop
Array index variables to access different elements of an array
ASCII number codes for keyboard characters
Bits and bytes have integer components to keep track of the location or size of an object

Geology
Elevation changes- how steep – change in height over distance; and then vertical exaggeration- how exaggerated is the slope when represented?
Counts of particular things (forams in a thin section, sheath folds in a mountain range, isoclinals folds and normal faults on a roadside outcrop)

Appendix J. Box Plots Comparing Student and Teacher CLES Survey Responses.

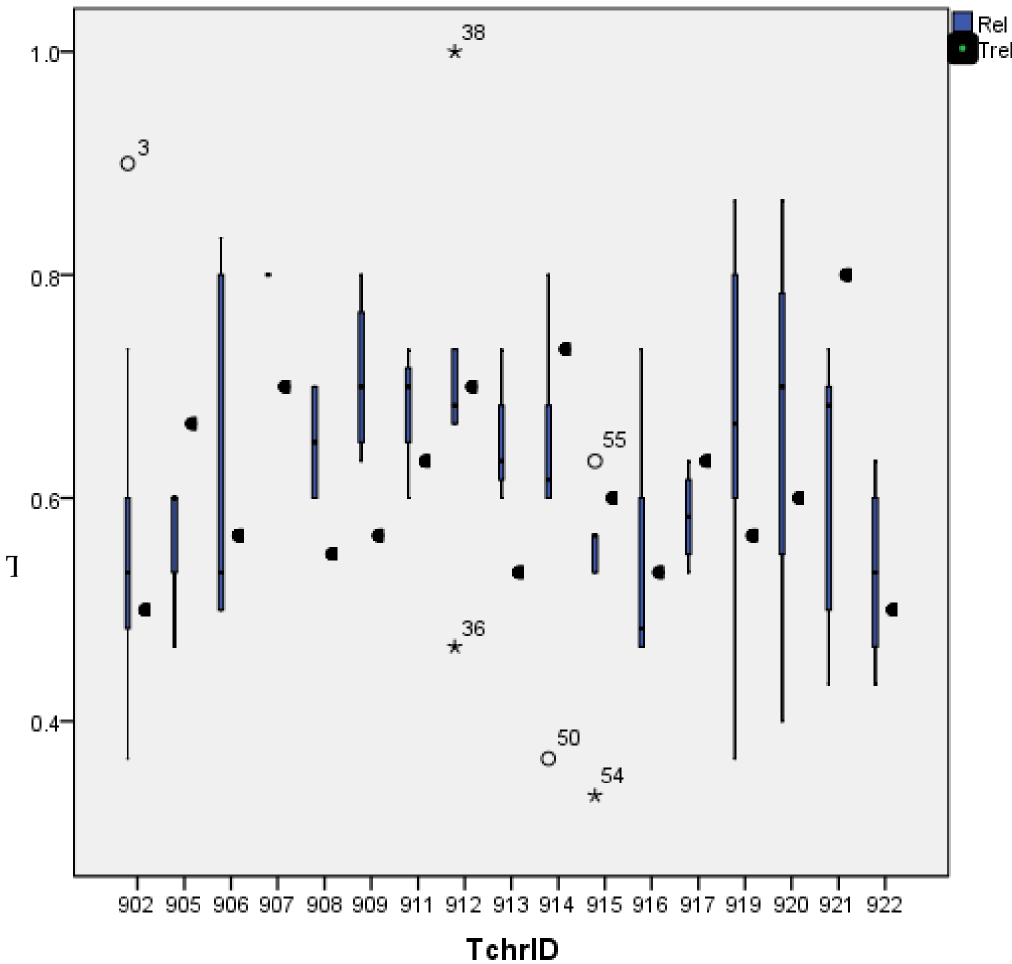


Figure 9 *Box Plot of Teacher and Student Ratings of Relevance on the CLES Survey*

Table 33 Percentage of Teachers with Similar Ratings as students: Relevance

Variable (n=17 teachers)	**Similar %**	**Higher %**	**Lower %**
Relevance	35	30	35

Source: Box Plot of Teacher and Student CLES ratings of Relevance

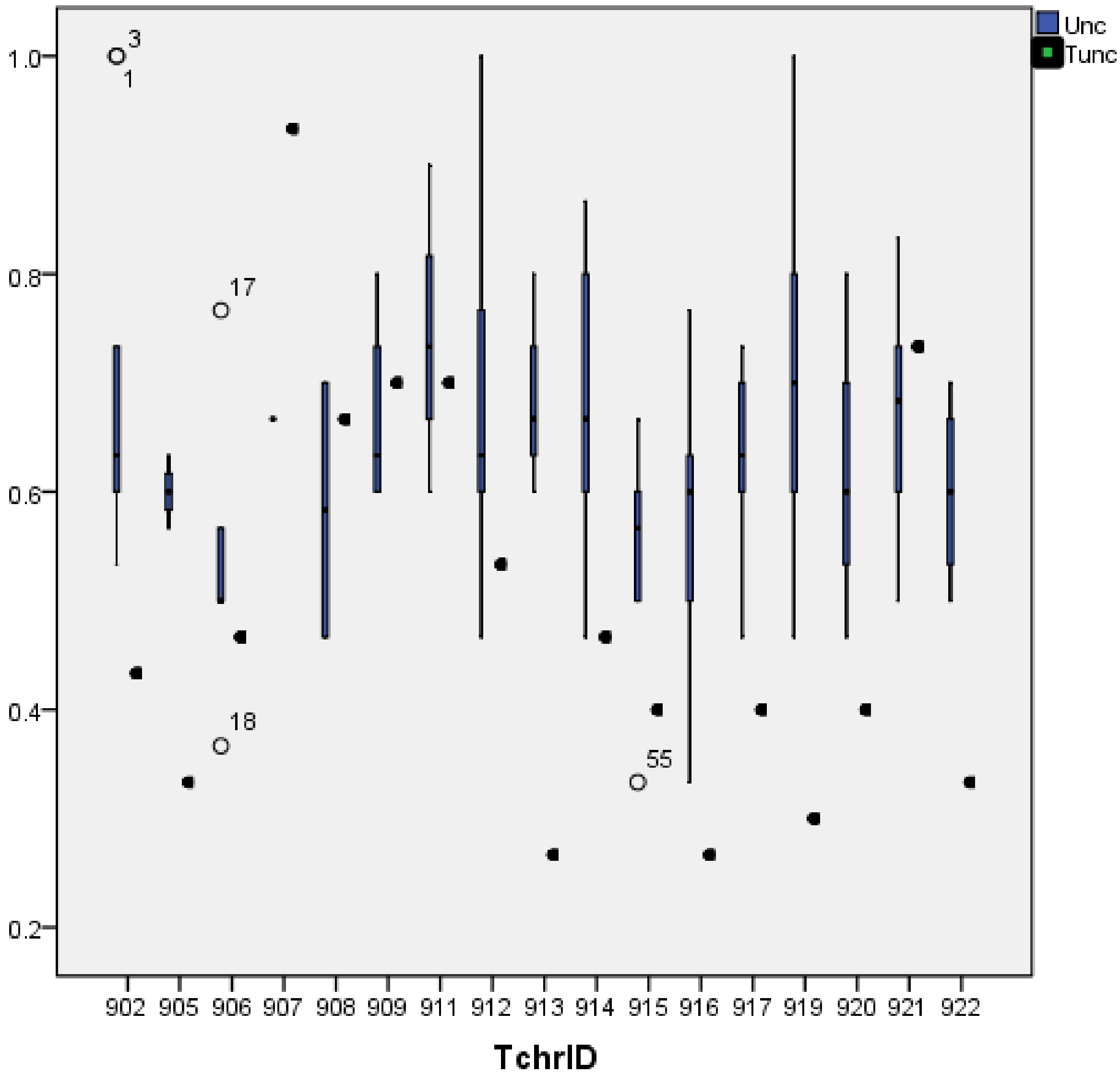


Figure 10   Box Plot of Teacher and Student Ratings of Uncertainty on the CLES Survey

Table 34   Percentage of Teachers with Similar Ratings as students on Uncertainty

Variable (n=17 teachers)	**Similar %**	**Higher %**	**Lower %**
Uncertainty	24	6	70

Source: Box Plot of Teacher and Student CLES ratings of Uncertainty

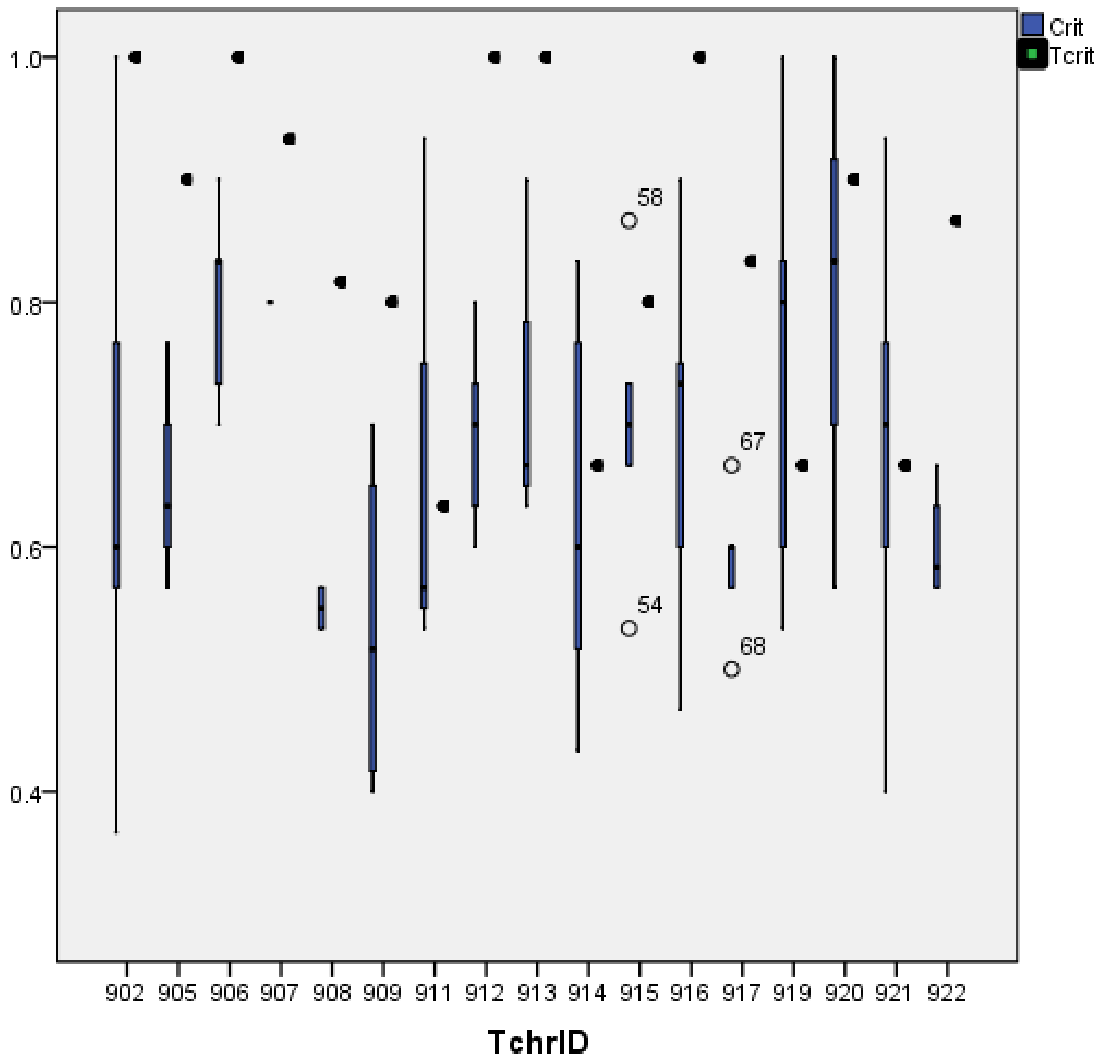


Figure 11   Box Plot of Teacher and Student Ratings of Critical Voice on the CLES

Table 35   Percentage of Teachers with Similar Ratings as students on Critical Voice

Variable (n=17 teachers)	**Similar %**	**Higher %**	**Lower %**
Critical Voice	29	71	0

Source: Box Plot of Teacher and Student CLES ratings of Critical Voice

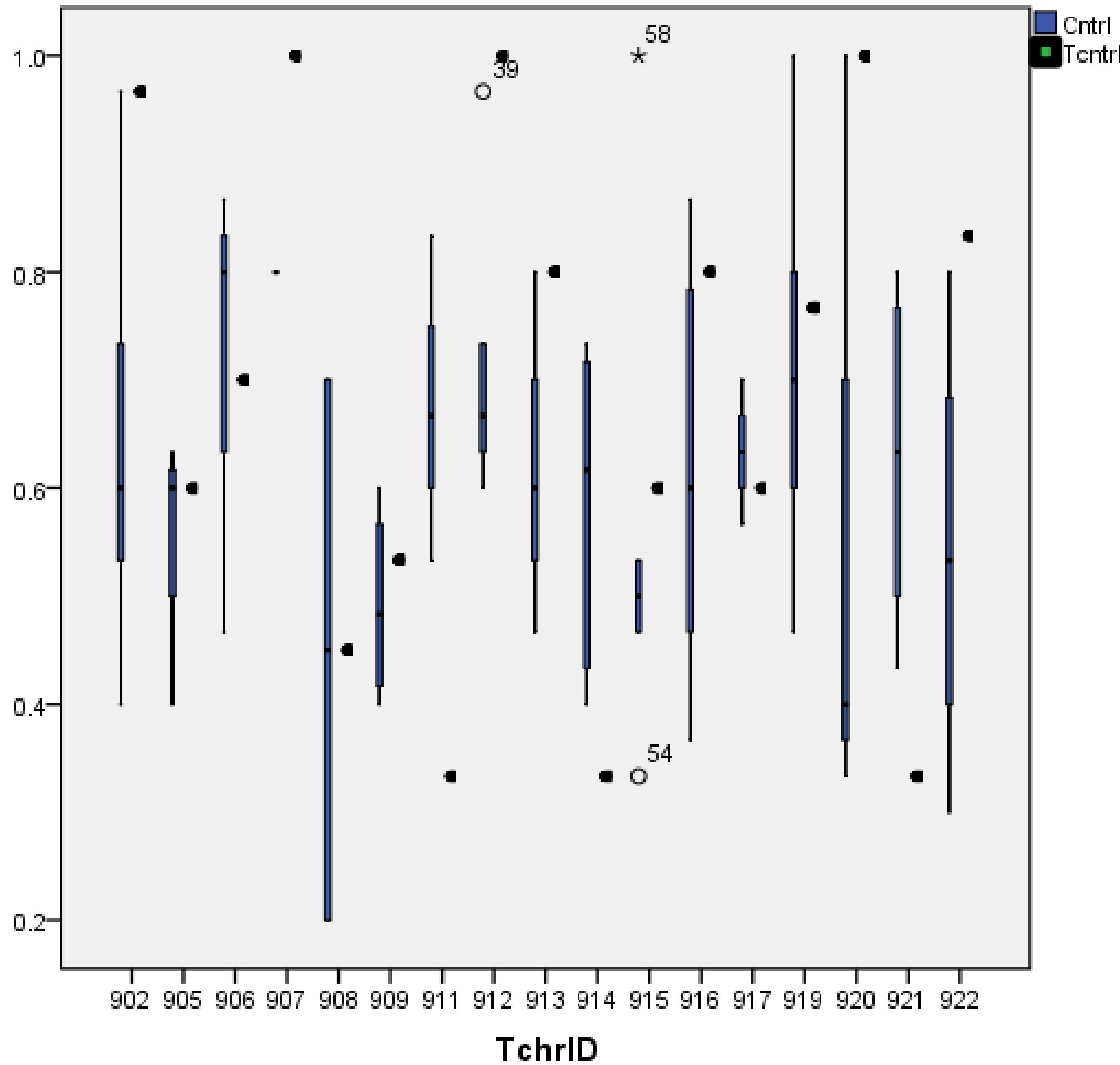


Figure 12   Box Plot of Teacher and Student Ratings of Shared Control on the CLES

Table 36   Percentage of Teachers with Similar Ratings as students on Shared Control

Variable (n=17 teachers)	**Similar %**	**Higher %**	**Lower %**
Shared Control	35	47	18

Source: Box Plot of Teacher and Student CLES ratings of Shared Control

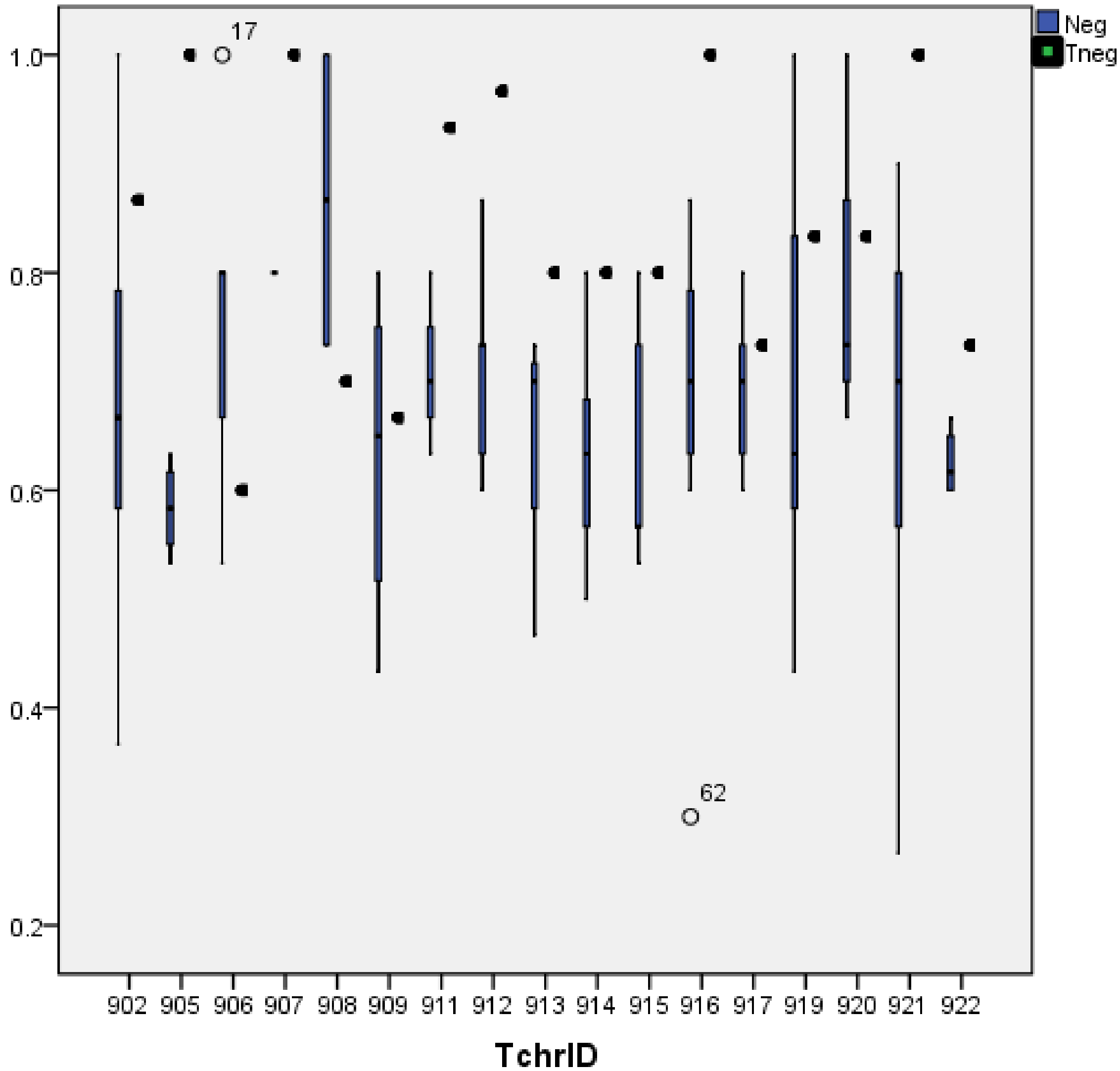


Figure 13   Box Plot of Teacher and Student Ratings of Negotiation on the CLES Survey

Table 37   Percentage of Teachers with Similar Ratings as students on Negotiation

Variable (n=17 teachers)	**Similar %**	**Higher %**	**Lower %**
Negotiation	23	65	12

Source: Box Plot of Teacher and Student CLES ratings of Negotiation

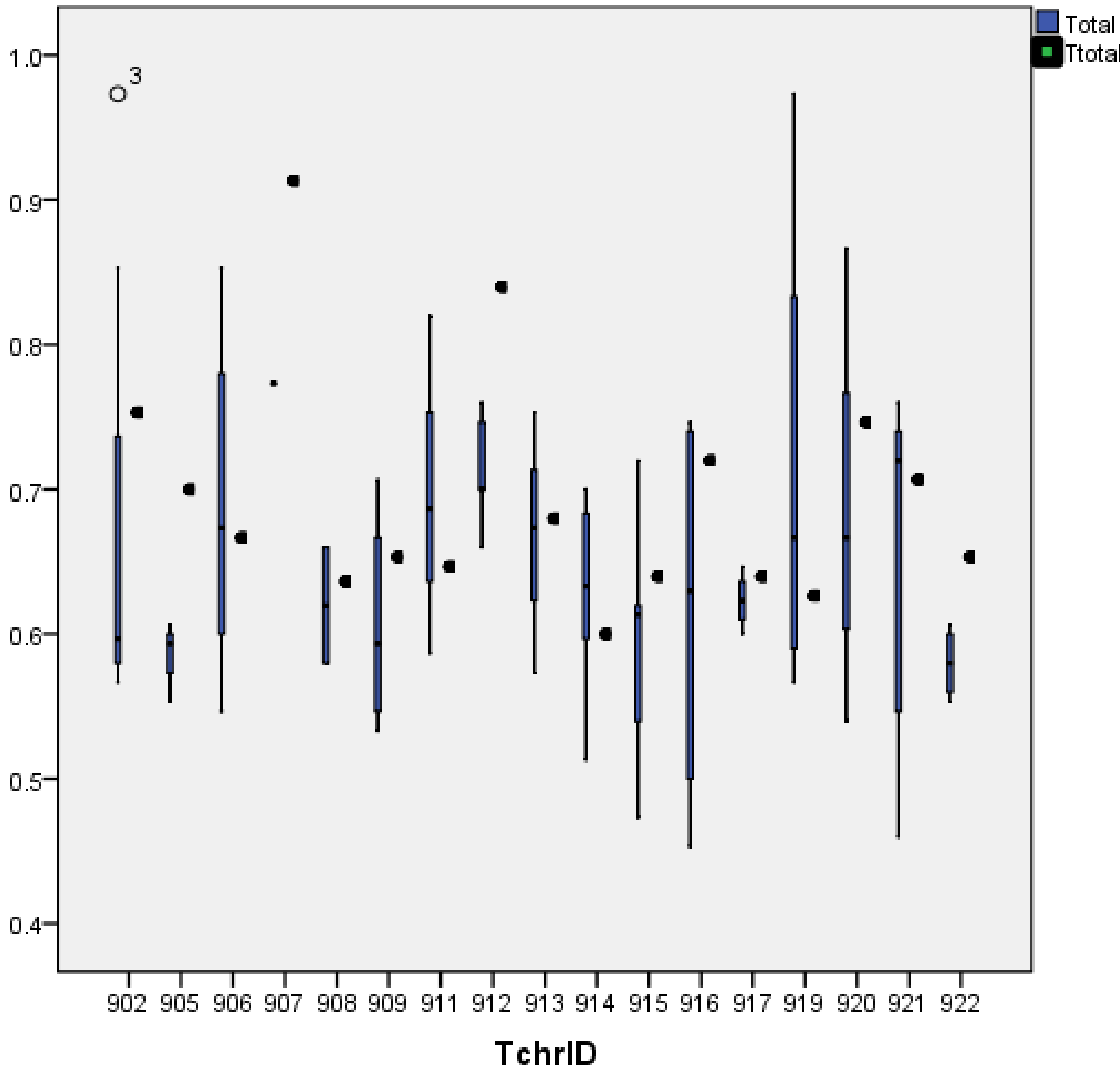


Figure 14 Box Plot of Teacher and Student Ratings on the CLES Survey Overall

Table 38 Percentage of Teachers with Similar Ratings as students Overall

Variable (n=17 teachers)	**Similar %**	**Higher %**	**Lower %**
Total	59	41	0

Source: Box Plot of Teacher and Student CLES ratings Overall on Total Score

Appendix K. Statistically Significant Correlations

Association	Correlation	Significance Level
Age & Teacher's Years Experience	.632	$p < .01$
Teachers' Years Experience & Advanced Degree	.588	$p < .01$
Prior Jumpstart Teacher & Teacher PCK posttest	-.520	$p < .05$
Teacher Major Math & Teacher CLES Relevance factor	.524	$p < .05$
Teacher CLES Relevance & Teacher CLES Uncertainty factor	.436	$p < .05$
Teacher CLES Relevance & Teacher CLES Critical	.446	$p < .05$
Teacher CLES Critical & Teacher CLES Control	.700	$p < .01$
Teacher PCK pretest & Teacher CLES Negotiation	-.484	$p < .05$
Teacher PCK CLES Total vs. CLES Control	.737	$p < .01$
Teacher PCK CLES Total vs. CLES Negotiation	.689	$p < .01$
Student Average Pretest(20) vs. Teacher Age	-.479	$p < .05$
Student Average Posttest Average (20) vs. Prior Jumpstart Exp.	-.511	$p < .01$
Student Average Pretest(25) vs. Teacher Age	-.479	$p < .05$
Student Average Pretest(25)vs. Post-test Average (20)	.992	$p < .001$
Student Average Posttest(25) vs. Prior Jumpstart Experience	-.491	$p < .05$
Student Posttest Avg. (25) vs. Student Pretest Avg. (20)	.994	$p < .001$

Note: Source: Author's Analysis of Teacher and Student Data
Note: Avg. (20) is the student average of the first 20 questions on the assessments. Avg. (25) is the student average on the complete 25 item assessment. Items 21-25 are extension problems.

Appendix L. Correlation Matrix- Teacher/Classroom Level

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	Teacher Age	1.0																	
2	Gender	0	1.0																
3	Teacher Yrs Experience	.632 **	.315	1.0															
4	Prior Jumpstart Teacher	.298	-.257	.313	1.0														
5	Advanced Degree	.252	.423	.588 **	.236	1.0													
6	Teacher Major Math	-.126	.168	.042	.281	.397	1.0												
7	Teacher PCK (pre)	.303	-.069	.317	-.097	-.026	N/A	1.0											
8	Teacher PCK (post)	.037	-.147	-.218	-.520 *	-.256	-.305	.268	1.0										
9	Teacher CLES Relevance	-.053	-.146	-.243	.119	-.078	.524*	-.358	-.312	1.0									
10	Teacher CLES Uncertainty	.059	.344	.103	-.047	.011	.295	-.343	-.172	.436 *	1.0								
11	Teacher CLES Critical	.166	-.184	-.070	.229	-.162	-.327	-.100	.346	-.446 *	-.270	1.0							
12	Teacher CLES Control	.278	-.104	-.011	.099	.206	-.309	-.284	.291	-.247	-.248	.700 **	1.0						
13	Teacher CLES Negotiation	-.099	-.023	-.172	.193	.175	.291	-.484 *	-.209	.401	-.016	.062	.359	1.0					
14	Teacher CLES-%	.166	0	-.110	.196	.125	.083	-.557	.044	.288	.291	.487*	.737 **	.689 **	1.0				
15	StdPreAvg20	-.479 *	.075	-.219	.194	.197	.094	.094	-.023	-.054	-.276	.011	-.092	.134	-.112	1.0			
16	StdPostAvg20	-.092	.361	-.178	- .511*	.100	.015	.073	.448 *	-.011	.068	.204	.321	.108	.310	.28 6	1.0		
17	StdPreAvg25	-.479 *	.015	-.248	.241	.162	.144	.087	-.218	-.013	-.252	.003	-.103	.141	-.098	.992 ***	.272	1.0	
18	StdPostAvg25	-.129	.327	-.224	-.491*	.089	.053	.020	.431	.054	.07	.190	.309	.160	.335	.296	.286	.994 ***	1

N=20 for Variables 1-5
N=18 for Variables

*correlation is significant at the .05 level (2-tailed))
**correlation is significant at the .01 level (2-tailed

Appendix M: Jumpstart 2010 Student Focus Group Interviews

Transcription Conducted by Sarah Harris 09/18/10

Transcription Codes:

S: Graduate Student who conducted the interviews

M#: Male Student (# 1 means the first male in the group to speak, #2 is the second)

F#: Female Student (# 1 means the first female in the group to speak, #2 is the second)

Interview 1

1 S: Can you tell me about what you have been doing with +/- numbers
2 M1: Use calculators, program to find negative and positive numbers
3 F1: make graphs, negative numbers
4 M1: compare, find the difference, and use equations, -5 + something equal something, and you have find what it equal in the calculator, so we have to write down every single step.
5 S: how do you use the calculator to find that
6 F1: if they give us and equation 5 + x + 2, put it into Y=
7 M1: to find the graph, Y= type in equation, press 2nd graph or just graph
8 F1: If we can’t see the graph we can go to WINDOW, I hadn’t learned that
9 M1: it is kind of complicated. Window is complicated, that is where you set the whole graph, Ymin, Xmax is complicated.
10 F1: When I first came here it was all day, I didn’t know how to use the calculator, and the teacher said you are going to have a pretest and I said what do you mean a pretest. I don’t know how to use the calculator. Then the teacher said, I just want to know what you know, and I said ok. Then when they gave me the pretest, I was like, dang, I don’t know nothing.
11 M1: I didn’t know anything
12 F1: fractions I didn’t know. Then they gave us a post-test. First we started practicing everything, I didn’t know how to do it first, now I can. I think I am ready to go to high school. I had a few things I didn’t know.
13 S: did you like the pretest and post test?

14 M1: It was the same test. Once you took it the first time it was a lot of stress, but now that the weeks went by and you learned all these things the post-test you actually recognize you know the stuff.
15 S: Did you use a number line to solve the problems
16 F1: the teacher puts up a number line so we started doing those things.
17 M1: like this start at the negative and then go positive
18 S: Turn to a blank page in your notebook, show me how you write it out.
19 M1 : regular number line, 0, 2, 4, 6, 8, 10. It is supposed to start at -6 add 4, 1, 2,3, 4 so -2.
20 F1: In these graphs, up and down, down is negative and up is positive so when you are talking about temperature or something or water.
21 M1. They make us do this to help us understand negative and positive and the direction they go in. The calculators had a program that lets us see a number line. You type in the equation and it shows you the arrows and which way it is going.
22 S: Do you remember what it is called
23 M1. Number line
24 F1. When I was first, I didn't know how to multiply positive and negative numbers. Sometimes when adding or multiplying, like when they say adding 5 and they say -5 you have to change the sign so I was confused. Say it is 5 +
25 M1. Suppose -5 x , or it is an equation
26 F1. Like the 6 is negative you have to change it so you are not subtracting, you are adding, so it makes it positive.
27 M1. If it is negative 6, instead of subtracting you add. You do the opposite of what they are asking you to do.
28 S. Can you show me 5 – 6 or 5 + (-6) is what you have.
29 M1. She always tells us to do a story right. So you owe someone 6 dollars but you only have 5. So you subtract, I still do the subtraction so its 6 minus 5 or 5 minus 6 which is negative one.
30 F1. Sometimes I get confused about multiplying positive and negative numbers. The teachers showed us about pieman. Pieman goes like something like that. A positive times a negative number , a negative and negative has to be a positive.
31 M1. I got confused on the algebra expressions because sometimes it says negative and I thought oh, you subtract it. You have to change that to a positive because two negative signs can't be together. So I just change that to a positive.
32 S. Do you guys like the number line
33 F1. I like the number line better
34 M1. I understand better with the pieman than the number line. The number line is too long, too many numbers.
35 F1. So if you have numbers like 100, 500, it doesn't go all that way. +5, +5, I get confused
36 M1. Like when a negative and a positive is a negative
37 S. How would you improve the number line activities
38 F1. Only use it for little numbers

39 M1. Don’t use it for extremely large numbers. If you have hard ones, I don’t want to cheat and use the calculator, so maybe use pieman or try to think and figure it out for yourself, or you could make a number line and use 100, 200 that is what I did.

40 S. how would you do this 5 – (120)

41 F1. That would be 115 by subtracting 5 from 120

42 M1. 20 minus 5 is 15, so it is 115. Like 4 x 5 is 20, so take away a 5 is 115.

43 S. how did you know there was 100.

44 M1. I just ignored the 1 that was for 100 and subtracted 5 from 20, and then added the one back on at the end.

Interview 2

1 S: Tell me what you have been learning about positive and negative numbers
2 F1: Subtracting and adding with them
3 F2: multiplying and dividing
4 S: can you tell me a little more about subtracting and adding with them
5 F1: We aren't the best at it but we are all getting better at it. I can't think right now because of the recorder
6 S: pretend it is not here, your teacher won't listen to them, this is a safe zone. What do you think about using a number line
7 F2: That was easy
8 F1: that made work a lot easier, because you showed your work and you could go back and check your work.
9 S: What do you mean?
10 F1: Sometimes we don't know how to show it, but if you use a numberline and it says -2 you can go to -2 and then if it says add 6 you can add 6 and see what you get. To me seeing something makes it easier.
11 F2: It was easy.
12 F3. Easy, helped you get the answer easier. You just have to put the numbers and get the answer so its easy.
13 S: Do you like it better than another method?
14 F3: Yeah, to see the numbers and to see which is the answer. If you put a -4 + -5 it is hard to see the numbers, but I put a number line and it is easier.
15 S: What about other methods?
16 F1: I don't know what its called, but it is like a chart and there is a negative side and a positive side, she (teacher) uses it a lot though.
17 F2: I forgot what it is called.
18 F3. Negative 6 you put it here and then if they say add something you put it on the positive side. So you can see what crosses each other out. So whichever has ones left over is your answer.
19 S: So if you had -6 and +5 how would you do it with that method.
20 F2: Adding or subtracting?
21 S: Let's try adding first.
22 F3. She's better at it, I get all confused.
23 S: -6 + 5, why don't you try the method you like the most with that
24 F2: Negative 1.
25 S: So how do you know its negative 1
26 F3: There's one left (points)
27 S: There's one left on the negative side? Ok what do you like least about the number line
28 F2. Sometimes you don't want to fill in all the negative sides, it takes a long time to write the number line.

29 F3. I like it, nothing bad about it.
30 S: Calculator activities graphing integers as vectors
31 F1: I don't like them, the way they explain it is complicated. I'd rather show my work on paper than use the calculator. Like when they tell you to press Y=, Zoom, window
32 F2: On the calculator there is a lot to do. Writing takes longer. I know the calculator is more accurate, but when she gives us the packet with all these instructions it is confusing.
33 F3. The calculator is easy. But pressing all the buttons is confusing, pressing delete, gets confusing.
34 S: If you had to rank the number line, the crossing out method, and the calculator
35 F1: the number line, her way (teacher), then the calculator
36 F2. The number line, then the calculator, then the teacher's way
37 F3. The number line, the teacher's way, then the calculator
38 S: suggestions to improve number line activities to make it better
39 F1: No
40 F2: Not really
41 F3: no
42 S: you said you had an issue with putting all the negative sings
43 F1: I guess you could just put a bracket with a negative above it. Because when you are taking a test you only have a certain amount of time, so you could just put a negative with a bracket above the number line
44 S: 5 – 12 This is not a test, we are just interested in how you think about it.
45 F1: I got a different answer so now I'm confused.
46 S: It's okay if you get stuck
47 F2. Positive side and then add 12 so its going to be 17
48 F1: I just subtracted, and it went to -7 rather than 17 so now I'm confused.
49 S: Is there anything you can do to check to see what's the right one
50 F1: I think its 17. If you are really just subtracting then it is -7, but if you are adding then it is 17.
51 F3. No this way over here.
52 S: do you want to try it another way, we don't have a calculator which is your second favorite.
53 F3. It's 17.
54 F1. That way (teacher way) is confusing to me. It's -7 (uses cell phone)
55 F1. I was right, okay, I'm not confused anymore. I just looked at 5 and subtracted 12 and got -7.

Interview 3

1 S: So tell me what you have been learning about positive and negative numbers
2 M1: we've just been learning to multiply, add, subtract, divide.
3 S: how were you doing that
4 F1: with a pencil, your mind, with the numbers, the teacher helps us
5 M1: I understand it, its' pretty easy. The teacher explained it more better than our middle school teacher.
6 S: Did you like the number line
7 F1: putting on the negative numbers and positive
8 F2: She is saying that, but you know how the negative numbers are here and the positive numbers are here and you have to count backwards.
9 S: Can you show me an example. What did you like most about that method
10 F1: everything.
11 S: Is there anything you didn't like
12 M1: I didn't like the number line. -3 + 10 - #, you have to keep going back and forth and back and forth . -3 , 1,2,3,4,5,6,7,8,9, 10. Seven.
13 S: Can you write out in a number sentence what you mean.
14 M1. I get this, but I don't like using that method. That little think shows these two will be negative and these two will be positive, so if you have a -3 and a – 26 you put those two together and add so it will be 3 + 26 you take those two negatives and put them as positive.
15 S: Can you show me how you would do this one with that method
16 M1: negative and a positive, 3 and -10 so it will be negative unless the number is big, so it would be a negative 7, but since 3 is less than 10 it will be positive.
17 S: What is your favorite thing? What about the graphing calculator activities where you graph the integers as vectors
18 F1: I don't know.
19 S: What was one of your favorite activities or methods for solving the problems
20 F1: (speaks to friend in Spanish). I don't know how to start
21 F2: look at patterns, find 1 go to that 1
22 M1: not the patterns we're talking about negatives
23 F2: okay never mind.
24 F1: add and multiply, you know the one we were doing with x, 4 + -2 and you have to cross it out. 1 ,2, 3, 4, put the positives here and negatives here and get the answer. 4 x 3 = 1, 2, 3, 4, and that's 12.

+ + + +
+ + + +
+ + + +

25 M1. Since this is a row, this is three positives
26 F1. No you don't do that, first you do the negatives then you do the positives
27 M1. Oh the opposites

28 F2, I don’t know how.
29 S: we talked about numberline, calculator, and this method of signs which did you like better
30 M1: This is the same but it is multiplying, 4 + 3 and 4 x 3 (both with signs)
31 F1. We like this method (signs), we learned this today about 9:00.
32 F2. I liked everything.
33 M1. We learned something else, too. 8x + 5 = 4x. Can we bring the book and show you.
34 S: It is about time to go.

Interview 4

1 S: Tell me a little bit about the positive and negative numbers activities you've been doing in class

2 M1. How a negative times a negative and stuff like that, how a positive and a negative react to each other. How they work.

3 S: can you show me or describe it

4 M1. -5 x 3, you can do 3 rows of or 5 rows or 3 rows of -5 and then that will equal -15 but if you turn it the other way around it will be a positive 15.

5 F1. You turn it 3 x -5

6 M1. +3 x -5 would equal +15 because a positive times a negative,

7 F1. A Positive times a negative equals a positive and a negative times a positive is a negative

8 M1. No a negative times a negative is a positive and a positive times a positive is a positive.

9 (writes out rules)

10 (Interruption – class going out to computer lab, noisy)

11 S: How did you come up with yours.

12 M1. A negative times a positive is a negative, a positive times a negative is a negative,

13 F1. A negative times a negative is a positive, and a negative times a positive is a negative

14 M1. And two negatives times a negative is a positive.

15 S: how would that help you on this problem

16 M1. A positive times a negative is a negative 3 x -5 = -15

17 S: Tell me what you liked most about using a number line

18 M1: It helps us if it is decimals it helps me.

19 S: What do you mean

20 M1. Like point, say there are ten lines, this is .10 and this is -.10 out of a whole. If there were only 5 lines it would be .20 and -.20.

21 S: So how would you solve a problem using this

22 F1. Like -25 – 5, so this is negative side and this is the positive side, so it is like you owe someone 25 dollars. Then you are asking for 5 more so you are going to add it up, so it is negative 30.

23 M1. But if you subtract it , it goes more to the

24 F1. When you are adding you go to the positive side, when you are subtracting you go to this side.

25 F1. -25 – 5 equal -20.

26 M1. That one is -20 and that one is -30. Hers says I owe you -25 but now you don't have to owe me 5 of the dollars. Mine says I've been buying you lunch so you owe me 25 dollars and now today you borrowed another 5 so now you owe me 30 dollars.

27 S: you explained it differently.

28 F1. If you have 25 and you take away 5 you are still asking for more. So you can be taking away, it depends on this number, if it is negative 5 you are going to go this way, you are going to subtract more, if it is positive 5 you are taking away so you are going to the positive side.

29 S: So you are getting different answers. Is there another method you could use to check

30 M1: Calculator

31 S: pushing the numbers or like you did in the class

32 F1. If you got a different answer, the negative sign has to be in the right spot in a calculator or we still could get different answers.

33 S: Which did you like better

34 F1: I liked the face for the negative times a negative

35 M1: the number line for adding and subtracting

36 F1: For adding and subtracting the number line

37 M1. For positive and negative fractions I turned it into a decimal. Percents are the same as decimals. Once you turn a fraction into a decimal you can see how much to take out

38 S: Suggestions for improving number line activity

39 M1: Do more fractions to help people who have problems. Make papers for that person who is having trouble with fractions. Everyone has the same paper but different signs.

40 F1. I liked them, I like the way it works, it helps to find the difference of negatives and positives, also to find what is bigger like -5 and -25, we use which one is closer to zero.

41 M1. It can help you find the range, the difference of a number, what is in between.

42 S: 5-12

43 M1. I wouldn't use a number line, I would just subtract, since you can't do it, it goes into the negative.

44 S: what do you mean you can't do it

45 M1. I have to use a number line. You are at 5 and you take away 12, 1,2,3,4,5,6……12 so it would be right here, at -7.

46 S: You also got -7 by just using numbers.

47 M1. I solve with just regular numbers and explain with the number line

48 F1. I flip it, 12 – 5 and the difference is 7. I find out which is bigger, 12 is bigger, 5 is positive, I have to take more than 5 so I have to go to the negative side. So I have to take 5 and 7 more which is 12 and I end up in the negative side which is -7.

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